

[illegible][illegible]

# BOOK OF ABSTRACTS



Instituto  
Politécnico  
Portalegre





---

# BOOK OF ABSTRACTS

---

Instituto Politécnico de Beja  
October 27, 2017  
Beja - PORTUGAL



## Participating Institutions



## Scientific Research Centers



## Scientific Sponsor

International Journal of Applied Mathematics and Statistics (IJAMAS)



PSE-Produtos e Serviços de Estatística



## List of Participants

- A. Manuela Gonçalves (mneves@math.uminho.pt)
- Alberto Simões (asimoes@ubi.pt)
- Ana Conceição (aicdoisg@gmail.com)
- Ana Lebre (alebre@ipbeja.pt)
- António Carloto (acarloto@ipbeja.pt)
- Carla Santos (carla.santos@ipbeja.pt)
- Carlos Ramos (ccr@uevora.pt)
- Célia Nunes (celian@ubi.pt)
- Clara Grácio (mgracio@uevora.pt)
- Cristina Dias (cpsilvadias@gmail.com)
- Dadang Hamzah (dadang.amir12@gmail.com)
- Dina Mateus (dinamateus@ipt.pt)
- Dora Gomes (dsrp@fct.unl.pt)
- Fernando Carapau (flc@uevora.pt)
- Filomena Teodoro (maria.alves.teodoro@marinha.pt)
- Hayat Zouiten (hayat.zouiten@yahoo.fr)
- Henrique Pinho (hpinho@ipt.pt)
- Ilda Rodrigues (ilda@ubi.pt)
- Irene Rodrigues (ipr@uevora.pt)
- Isabel Malico (imbm@uevora.pt)
- João Barros (joao.barros@ipbeja.pt)
- João Branco (jbranco@math.ist.utl.pt)
- João Miranda (jlmiranda@estgp.pt)
- João Romacho (jromacho@estgp.pt)
- João Santos (joaof.santos@ipbeja.pt)
- João Portugal (jportugal@ipbeja.pt)
- José Pereira (lobopereiramail@gmail.com)
- José Saias (jsaias@uevora.pt)
- Juan Zapata (jgzapata@unex.es)
- Leonardo Andrade (andrade9653@gmail.com)
- Luís Bandeira (lmzb@uevora.pt)
- Luís Domingues (Luis.domingues@ipbeja.pt)
- Luís M. Grilo (lgrilo@ipt.pt)
- Manuel Alberto (d11837@alunos.uevora.pt)
- Manuel Branco (mbb@uevora.pt)
- M. Manuela Azevedo (mazevedo@ipbeja.pt)
- M. Teresa Godinho (mtgodinho@ipbeja.pt)
- M. Isabel Colaço (isabel.colaco@ipbeja.pt)
- Maria Varadinov (dinov@estgp.pt)



- 
- Mesbahi Oumaima (oumaimames@gmail.com)
  - Mouhaydine Tlemcani (tlem@uevora.pt)
  - Nuno Brites (brites@uevora.pt)
  - Nuno Beja (nbeja@ipbeja.pt)
  - Patrícia Filipe (pasf@uevora.pt)
  - Pedro Cravo (pedro.cravo@ipbeja.pt)
  - Pedro Silva (pedrosilva@ipbeja.pt)
  - Ramez Aldwihe (eng.ramez87@hotmail.com)
  - Rogério Serôdio (rserodio@ubi.pt)
  - Sara Fernandes (sal@uevora.pt)
  - Sellami Assia (sellami.assia@gmail.com)
  - Sérgio Costa (sergiocavaleirocosta@gmail.com)
  - Sofia Ramôa (sramoa@ipbeja.pt)
  - Susana Faria (sfaria@math.uminho.pt)
  - Teresa Oliveira (teresa.oliveira@uab.pt)
  - Zeferino Caxala (Zeferino.caxala@ubi.pt)

## **Committees**

### **Local Organizing Committee (IPBeja, Portugal)**

Carla Santos (Local Chair)  
Luís Domingues  
M. Manuela Azevedo  
M. Teresa Godinho  
Ana Lebre  
Pedro Cravo

### **Organizing Committee**

Carla Santos, Instituto Politécnico de Beja, Portugal (Local Chair)  
Cristina Dias, Instituto Politécnico de Portalegre, Portugal  
Fernando Carapau, Universidade de Évora, Portugal  
Luís M. Grilo, Instituto Politécnico de Tomar, Portugal (Chair)

### **Scientific Committee**

Ashwin Vaidya, Montclair State University, USA  
Arminda Manuela Gonçalves, Universidade do Minho, Portugal  
Ana Silvestre, Instituto Superior Técnico, Portugal  
Amílcar Oliveira, Universidade Aberta, Portugal  
Alexandra Moura, Instituto Superior de Economia e Gestão, Portugal  
Alberto Simões, Universidade da Beira Interior, Portugal  
Adelaide Figueiredo, Universidade do Porto, Portugal  
Aldina Correia, Instituto Politécnico do Porto, Portugal  
Bong Jae Chung, George Mason University, USA  
Carla Santos, Instituto Politécnico de Beja, Portugal  
Carlos Ramos, Universidade de Évora, Portugal  
Carlos Agra Coelho, Universidade Nova de Lisboa, Portugal  
Célia Nunes, Universidade da Beira Interior, Portugal  
Cristina Dias, Instituto Politécnico de Portalegre, Portugal  
Dário Ferreira, Universidade da Beira Interior, Portugal  
Delfim F. M. Torres, Universidade de Aveiro, Portugal  
Dora Gomes, Universidade Nova de Lisboa, Portugal  
Eliaana Costa e Silva, Instituto Politécnico do Porto, Portugal  
Fernanda Figueiredo, Universidade do Porto, Portugal  
Filipe Marques, Universidade Nova de Lisboa, Portugal  
Fernando Carapau, Universidade de Évora, Portugal  
Filomena Teodoro, Escola Naval, Portugal  
Gabriel Pires, Instituto Politécnico de Tomar, Portugal

Hermenegildo Oliveira, Universidade do Algarve, Portugal  
Henrique Pinho, Instituto Politécnico de Tomar, Portugal  
Irene Rodrigues, Universidade de Évora, Portugal  
João Janela, Instituto Superior de Economia e Gestão, Portugal  
José Augusto Ferreira, Universidade de Coimbra, Portugal  
João Romacho, Instituto Politécnico de Portalegre, Portugal  
João Luís Miranda, Instituto Politécnico de Portalegre, Portugal  
Lígia Ferreira, Universidade de Évora, Portugal  
Luís Bandeira, Universidade de Évora, Portugal  
Luís M. Grilo, Instituto Politécnico de Tomar, Portugal  
Maria de Fátima de Almeida Ferreira, Inst. Polit. do Porto, Portugal  
Maria Luísa Morgado, Uni. de Trás-os-Montes e Alto Douro, Portugal  
Manuel Branco, Universidade de Évora, Portugal  
Milan Sthelík, University Linz, Austria  
Mouhaydine Tlemcani, Universidade de Évora, Portugal  
Paulo Correia, Universidade de Évora, Portugal  
Pedro Lima, Instituto Superior Técnico, Portugal  
Russell Alpizar-Jara, Universidade de Évora, Portugal  
Rafael Santos, Universidade do Algarve, Portugal  
Sara Fernandes, Universidade de Évora, Portugal  
Sandra Ferreira, Universidade de Beira Interior, Portugal  
Sérgio Nunes, Instituto Politécnico de Tomar, Portugal  
Susana Faria, Universidade do Minho, Portugal  
Tanuja Srivastava, Indian Institute of Technology, India  
Teresa Oliveira, Universidade Aberta, Portugal  
Valter Vairinhos, Centro de Investigação Naval, Portugal

## Technical Specifications

**Title:**

IV Workshop on Computational Data Analysis and Numerical Methods -  
Book of Abstracts

**Web page:**

<http://www.wcdanm-beja17.uevora.pt/>

**Editor:**

Instituto Politécnico de Beja  
Rua Pedro Soares, Campus do Instituto Politécnico de Beja  
Apartado 6155  
7800-295 Beja, Portugal

**Workshop place:**

Escola Superior de Tecnologia e Gestão do Instituto Politécnico de Beja

**Authors:**

Luís M. Grilo, Fernando Carapau, Carla Santos and Cristina Dias

**Published and printed by:**

UE-Universidade de Évora

Copyright © 2017 left to the authors of individual papers

All rights reserved

**ISBN:** 978-989-8008-28-2 (print version), 978-989-8008-29-9 (online version)

## Preface

Dear participants, colleagues and friends,

WELCOME TO THE IV WCDANM 2017, it is a great honour and a privilege to give you all our warmest welcome to the fourth Annual Workshop of Computational Data Analysis and Numerical Methods (IV WCDANM).

This Workshop is being held at the beautiful campus of Instituto Politécnico de Beja, located in the city of Beja, Portugal. The host institution, has been fully committed on this challenge from the beginning, and we do hope that the final result exceed expectations for participants, sponsors and organizers. We wish to thank specially to them, as this event could not be possible without any of these essential parts.

The support from sponsors, the availability and contributions from Invited speakers, the high scientific level of oral and poster presentations from participants and, at the end, curious, active and interested assistants, will contribute to the success of the meeting since it is a free fee meeting. From the organizing committee we want also to thank them for their continuous help and implication in the effort. Finally, our gratitude to the members of the scientific and organizing committees that have been working together hard to yield a balanced, wide-scoped and interesting programme. Special thanks to the Local Chair Carla Santos (Instituto Politécnico de Beja), Cristina Dias (Instituto Politécnico de Portalegre) and Fernando Carapau (Universidade de Évora), who have been in charge of many tasks, and have fulfilled a brilliant labour.

As in previous meetings, Computational Data Analysis and Numerical Methods will be approached by recognized experts in specific fields. The CDANM Workshop series is unique in that it brings together researchers from all over the country who use Data analysis and Numerical methods in their research with particular interest in applications.

For the first time authors have the opportunity to publish their papers in a special issue of the International Journal of Applied Mathematics and



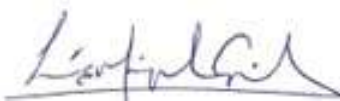
Statistics (IJAMAS), after refereeing process and according to the conditions of IJAMAS.

We hope that you enjoy the Workshop and find it intellectually stimulating. We wish that it could provide an opportunity for the mathematical community to work together and to plan new initiatives.

We are very happy you have joined us in Beja and hope you have a memorable time!

Beja, October 27th, 2017

Luís Miguel Grilo



Chair of the WCDANM  
Instituto Politécnico de Tomar

## Programme



**IV WCDANM, Instituto Politécnico de Beja  
October 27, 2017, Portugal**

**Event Place: Escola Superior de Tecnologia e Gestão do Instituto Politécnico  
de Beja**

09:00-09:30 Registration

09:30-10:00 Open Ceremony of the IV WCDANM17

10:00-10:25 Contributed Talk (25m apresentation and 5m discussion)

Room A1 (Chair: Luís M. Grilo):

Cristina Dias, J. Miranda, C. Santos and J. T. Mexia (**Anova Analysis and  
Related Techniques for Structured Families of Symmetric Stochastic Matrices**)

Room A2 (Chair: Teresa Oliveira):

J.F. Santos, M. M. Portela and I. Pulido-Calvo (**Regional Frequency Analysis  
of Extreme Climate Phenomena**)

Room S1 (Chair: Fernando Carapau):

Luís Bandeira and Carlos C. Ramos (**Oscillators on a Cantor set**)

10:30-10:55 Contributed Talk (25m apresentation and 5m discussion)

Room A1 (Chair: Luís M. Grilo):

Célia Nunes, G. Capistrano, D. Ferreira, S. S. Ferreira and J.T. Mexia (**Ran-  
dom sample sizes in one-way ANOVA with fixed effects**)

Room A2 (Chair: Teresa Oliveira):

João A. Branco and Ana M. Pires (**Whos not afraid of Big Data?**)

Room S1 (Chair: Fernando Carapau):

Carlos C. Ramos (**Nonlinearity and control**)

11:00-11:30 Coffee Break and Posters Session

11:30-11:55 Contributed Talk (25m apresentation and 5m discussion)

Room A1 (Chair: Luís M. Grilo):

Cristina Dias, João Romacho and Maria J. Varadinov (**Risk behaviour in the asset management industry**)

Room A2 (Chair: Teresa Oliveira):

Isabel Colaço (**The Stern-Brocot tree**)

Room S1 (Chair: Fernando Carapau):

M. Filomena Teodoro (**Numerical Schemes to Solve Some MTFDE's**)

12:00-12:25 Contributed Talk (25m apresentation and 5m discussion)

Room A1 (Chair: Luís M. Grilo):

Luís M. Grilo and Helena L. Grilo (**Statistical agreement between two methods of measurement**)

Room A2 (Chair: Teresa Oliveira):

Teresa Oliveira and Amílcar Oliveira (**Exploring R features for Experimental Designs**)

Room S1 (Chair: Fernando Carapau):

Fernando Carapau, P. Correia, L. Grilo and R. Conceição (**Axisymmetric Motion of a Proposed Generalized Non-Newtonian Fluid Model with Shear-dependent Viscoelastic Effects**)

12:30-14:00 Lunch

14:00-14:25 Contributed Talk (25m apresentation and 5m discussion)

Room A1 (Chair: Cristina Dias):

Dora Prata Gomes and M. Neves (**Improving the Extremal Index Blocks Estimator**)



Room A2 (Chair: João A. Branco):

Luís Domingues and José G. Dias (**A comparison of the performance of restoration-classification models with spatial data**)

Room S1 (Chair: Carlos C. Ramos):

Assia Sellami, O. Mesbahi, K. kandoussi, R. Otmani, A. Hajjaji and M. Tlemçani (**The Modeling of a simple PV and Cooling PV panel using numerical methods**)

14:30-14:55 Contributed Talk (25m presentation and 5m discussion)

Room A1 (Chair: Cristina Dias):

Pedro M. Cravo (**Satisfaction with the Tourist Experience: An Applications of the Nonlinear Estimation Model**)

Room A2 (Chair: João A. Branco):

Nuno M. Brites and Carlos A. Braumann (**Comparison of fishing policies for populations with weak Allee effects in a random environment**)

Room S1 (Chair: Carlos C. Ramos):

Oumaima Mesbahi, A. Sellami, K. kandoussi, R. Otmani, A. Hajjaji and M. Tlemçani (**Coupling of numerical algorithms: An application to a nonlinear engineering model**)

15:00-15:25 Contributed Talk (25m presentation and 5m discussion)

Room A1 (Chair: Cristina Dias):

Dadang Amir Hamzah and Eugénio Rocha (**Homotopy Iterative Splitting Method to solve the generalized Fisher's Equation**)

Room A2 (Chair: João A. Branco):

João Miranda Cristina Dias and Maria J. Varadinov (**Training Young Researchers in Pharmaceutical Supply Chains-Medicines Shortages**)

Room S1 (Chair: Carlos C. Ramos):

Juan Luis García Zapata, Maria C. Grácio and I. Rodrigues (**Spectral Clustering Tools for Analysis of Learning Trajectories in the Student Network of ora University**)

15:30-15:55 Contributed Talk (25m apresentation and 5m discussion)

Room A1 (Chair: Cristina Dias):

Hayat Zouiten and Ali Boutoulout (**Regional enlarged observability for parabolic semi-linear systems**)

Room A2 (Chair: João A. Branco):

J.L. Pereira, L. Mendes and T.A. Oliveira (**Comparison of means through GLM - An example in Oral Health**)

Room S1 (Chair: Carlos C. Ramos):

Manuel B. Branco (**Some results on the Frobenius coin problem**)

16:00-16:25 Contributed Talk (25m apresentation and 5m discussion)

Room A1 (Chair: Cristina Dias):

Sérgio C. Costa, Fernando M. Janeiro and Isabel Malico (**A Genetic Algorithm tweak for result improvement in inverse optimization problems**)

Room S1 (Chair: Carlos C. Ramos):

L.P. Castro and A.M. Simões (**A New Type of Stability: semi-Hyers-Ulam-Rassias Stability**)

16:30-17:00 Coffee Break and Posters Session

### **Posters Authors**

Ana C. Conceição (**PDLs and Flowcharts in Operator Theory**)

Leonardo Andrade, Pedro Gonves, Mouhadydine Tlemçani and Fernando Janeiro (**Virtual Instrumentation: Evaluation of a data acquisition**)

António Carloto (**Wine quality ratings versus price in The Wine Enthusiast Magazine**)

Fernando Carapau, Paulo Correia and Luís M. Grilo (**One-dimensional Third-grade Fluid Model**)

Carla Santos, Célia Nunes, Cristina Dias and João Tiago Mexia (**Building up complex models with commutative orthogonal block structure**)

Carla Santos, Célia Nunes, Cristina Dias and João Tiago Mexia (**Condensing normal OBS**)

Cristina Dias, Carla Santos, João Romacho, Maria José Varadinov and João Tiago Mexia (**Symmetric Stochastic Matrices**)

M. Filomena Teodoro, Carla Simão, Margarida Abranches, Sofia Deuchande and Ana Teixeira (**Prevalence of Pediatric Hiypertension: a Preliminary Approach**)

Manuel Alberto, Dulce Gomes and Patrícia A. Filipe (**Trends and seasonality of the road accidents in Angola from 2002 to 2015**)

António Breda D´Azevedo and Ilda Inácio Rodrigues (**An overview of the classification of Bicontactual Hypermaps**)

A. Manuela Gonçalves and Andreia Ribeiro (**E-Commerce: a statistical approach for supply forecasting**)

Dina Mateus and Henrique Pinto (**Regression model of sugarcane juice sugar content as a function of the measurement height on the stalk**)

Nuno M. Brites, Pedro Melgueira, Irene Rodrigues and Lúcia Ferreira (**Application of data mining techniques to E-learning data**)

Ramez Aldwihe and José Sajas (**Computational vision applied in automotive driving support systems**)

J. Rodrigues, A.M. Gonçalves, S. Faria, A.R. Gomes and C. Simões (**The relations between work-family conflicts, burnout, and cognitive appraisal: a structural equation modelling**)

Luís Sancho, Victor Figueira and M. Teresa Godinho (**The Image of the Alentejo as a Tourist Destination through the eyes of Lisbon's inhabitants**)

Zeferino Caxala, Rogério Serôdio and Ilda Inácio Rodrigues (**Investigating the properties of Rascal's Triangle**)

Sofia Ramôa, Pedro Oliveira e Silva, Teresa Vasconcelos, Paulo Fortes and João Portugal (**Applying statistical methods on the analysis of ecological preferences of the spontaneous flora**)

Susana Faria and Luísa Novais (**Determining the Number of Components in Mixtures of Linear Mixed Models**)

Maria José Varadinov, Nicolau Almeida, João Romacho, Cristina Dias and Carla Santos (**Multivariate APC model in the analyze of the logistics activities of companies that implement or not a system of reverse logistics**)

17:00 Social Program Visit: Historic center of Beja

20:00 Conference Dinner (Restaurante A Pipa (Rua da Moeda 8, Beja))

# Contents

Participating Institutions .....	i
Scientific Research Centers .....	ii
Scientific Sponsor .....	iii
List of Participants .....	iv
Committees .....	vi
Technical Specifications .....	viii
Preface .....	ix
Programme .....	xi

---

## Invited Speakers

---

<b>Carlos C. Ramos</b> <i>Nonlinearity and control</i> .....	1
<b>Manuel B. Branco</b> <i>Some results on the Frobenius coin problem</i> .....	2
<b>Célia Nunes, Gilberto Capistrano, Dário Ferreira, Sandra S. Ferreira and João Tiago Mexia</b> <i>Random sample sizes in one-way ANOVA with fixed effects</i> .....	4
<b>Fernando Carapau, Paulo Correia, Luís M. Grilo and Ricardo Conceição</b> <i>Axisymmetric Motion of a Proposed Generalized Non-Newtonian Fluid Model with Shear-dependent Viscoelastic Effects</i> .....	6
<b>Luís M. Grilo and Helena L. Grilo</b> <i>Statistical agreement between two methods of measurement</i> .....	8
<b>Teresa A. Oliveira and Amílcar Oliveira</b> <i>Exploring R features for Experimental Designs</i> .....	10
<b>M. Filomena Teodoro</b> <i>Numerical Schemes to Solve Some MTFDE's</i> .....	11



---

## Contributed Talk

---

<b>Cristina Dias, João Miranda, Carla Santos and João Tiago Mexia</b> <i>Anova Analysis and Related Techniques for Structured Families of Symmetric Stochastic Matrices</i> .....	1
<b>J.F. Santos, M. M. Portela and I. Pulido-Calvo</b> <i>Regional Frequency Analysis of Extreme Climate Phenomena</i> .....	3
<b>Luís Bandeira and Carlos C. Ramos</b> <i>Oscillators on a Cantor set</i> .....	5
<b>João A. Branco and Ana M. Pires</b> <i>Whos not afraid of Big Data?</i> .....	6
<b>Cristina Dias, João Romacho and Maria José Varadinov</b> <i>Risk behaviour in the asset management industry</i> .....	7
<b>Isabel Colaço</b> <i>The Stern-Brocot tree</i> .....	9
<b>Dora Prata Gomes and Manuela Neves</b> <i>Improving the Extremal Index Blocks Estimator</i> .....	11
<b>Luís Filipe Domingues and José G. Dias</b> <i>A comparison of the performance of restoration-classification models with spatial data</i> .....	13
<b>Assia Sellami, Oumaima Mesbahi, Khalid kandoussi, Rabie El Otmani, Abdeloawahed Hajjaji and Mouhaydine Tlemçani</b> <i>The Modeling of a simple PV and Cooling PV panel using numerical methods</i> .	14
<b>Pedro M. Cravo</b> <i>Satisfaction with the Tourist Experience: An Applications of the Nonlinear Estimation Model</i> .....	15
<b>Nuno M. Brites and Carlos A. Braumann</b> <i>Comparison of fishing policies for populations with weak Allee effects in a random environment</i> .....	16
<b>Oumaima Mesbahi, Assia Sellami, Khalid kandoussi, Rabie El Otmani, Abdeloawahed Hajjaji and Mouhaydine Tlemçani</b> <i>Coupling of numerical algorithms: An application to a nonlinear engineering model</i>	18

<b>Dadang Amir Hamzah and Eugénio Rocha</b> <i>Homotopy Iterative Splitting Method to solve the generalized Fisher's Equation</i>	20
<b>João Miranda, Cristina Dias and Maria José Varadinov</b> <i>Training Young Researchers in "Pharmaceutical Supply Chains-Medicines Short-ages"</i>	21
<b>Juan Luis García Zapata, Maria Clara Grácio and Irene Rodrigues</b> <i>Spectral Clustering Tools for Analysis of Learning Trajectories in the Student Network of Évora University</i>	23
<b>Hayat Zouiten, Ali Boutoulout</b> <i>Regional enlarged observability for parabolic semi-linear systems</i>	25
<b>J.L. Pereira, L. Mendes and T.A. Oliveira</b> <i>Comparison of means through GLM - An example in Oral Health</i>	26
<b>Sérgio Cavaleiro Costa, Fernando M. Janeiro and Isabel Malico</b> <i>A Genetic Algorithm tweak for result improvement in inverse optimization problems</i>	28
<b>L.P. Castro and A.M. Simões</b> <i>A New Type of Stability: semi-Hyers-Ulam-Rassias Stability</i>	30
<hr/> <b>Contributed Poster</b> <hr/>	
<b>Luís Sancho, Victor Figueira and M. Teresa Godinho</b> <i>The Image of the Alentejo as a Tourist Destination through the eyes of Lisbon's inhabitants</i>	35
<b>Maria José Varadinov, Nicolau Almeida, João Romacho, Cristina Dias and Carla Santos</b> <i>Multivariate APC model in the analyze of the logistics activities of companies that implement or not a system of reverse logistics</i>	36
<b>Cristina Dias, Carlos Santos, João Romacho, Maria José Varadinov and João Tiago Mexia</b> <i>Symmetric Stochastic Matrices</i>	38
<b>Ana C. Conceição</b> <i>PDLs and Flowcharts in Operator Theory</i>	39
<b>Carla Santos, Célia Nunes, Cristina Dias and João Tiago Mexia</b> <i>Building up complex models with commutative orthogonal block structure</i>	40

<b>Carla Santos, Célia Nunes, Cristina Dias and João Tiago Mexia</b>	
<i>Condensing normal OBS</i> .....	43
<b>Dina Mateus and Henrique Pinho</b>	
<i>Regression model of sugarcane juice sugar content as a function of the measurement height on the stalk</i> .....	45
<b>Fernando Carapau, Paulo Correia and Luís M. Grilo</b>	
<i>One-dimensional Third-grade Fluid Model</i> .....	47
<b>Zeferino Caxala, Rogério Serôdio and Ilda Inácio Rodrigues</b>	
<i>Investigating the properties of Rascal's Triangle</i> .....	49
<b>M. Filomena Teodoro, Carla Simão, Margarida Abranches, Sofia Deuchande and Ana Teixeira</b>	
<i>Prevalence of Pediatric Hypertension: a Preliminary Approach</i> .....	50
<b>Sofia Ramôa, Pedro Oliveira e Silva, Teresa Vasconcelos, Paulo Fortes and João Portugal</b>	
<i>Applying statistical methods on the analysis of ecological preferences of the spontaneous flora</i> .....	52
<b>J. Rodrigues, A.M. Gonçalves, S. Faria, A.R. Gomes, C. Simões</b>	
<i>The relations between work-family conflicts, burnout, and cognitive appraisal: a structural equation modelling</i> .....	55
<b>Susana Faria and Luísa Novais</b>	
<i>Determining the Number of Components in Mixtures of Linear Mixed Models</i> .	56
<b>A. Manuela Gonçalves and Andreia Ribeiro</b>	
<i>E-Commerce: a statistical approach for supply forecasting</i> .....	57
<b>Leonardo Andrade, Pedro Gonçalves, Mouhaydine Tlemçani and Fernando Janeiro</b>	
<i>Virtual Instrumentation: Evaluation of a data acquisition</i> .....	58
<b>Ramez Aldwihe and José Saias</b>	
<i>Computational vision applied in automotive driving support systems</i> .....	59
<b>Nuno M. Brites, Pedro Melgueira, Irene Rodrigues and Lígia Ferreira</b>	
<i>Application of data mining techniques to E-learning data</i> .....	60
<b>António Carloto</b>	
<i>Wine quality ratings versus price in The Wine Enthusiast Magazine</i> .....	62

---

<b>António Breda D’Azevedo and Ilda Inácio Rodrigues</b>	
<i>An overview of the classification of Bicontactual Hypermaps</i> . . . . .	64
<b>Manuel Alberto, Dulce Gomes and Patrícia A. Filipe</b>	
<i>Trends and seasonality of the road accidents in Angola from 2002 to 2015</i> . . .	66
<b>Index of authors</b> . . . . .	69



## Invited Speakers



## Nonlinearity and control

Carlos C. Ramos<sup>1,2</sup>

<sup>1</sup>Universidade de Évora, Departamento de Matemática, Portugal

<sup>2</sup>Centro de Investigação em Matemática e Aplicações (CIMA), Portugal

**Corresponding Author:** *ccr@uevora.pt*

### Abstract

We present and discuss a class of dynamical system characterized by two maps,  $m$  and  $r$ : the map  $m$  is defined on a topological space and is characterized by a set of real parameters. The map  $r$ , called regulatory map, is defined on the space of the parameters of  $m$  and controls its dynamical behavior. This setting can be seen as an abstraction of the metabolism concept, and therefore  $m$  is called metabolic map. Certain optimization problems are naturally established for the pair  $(m, r)$ . For given parameters, we consider a maximizing function, which depends on the metabolic orbit during a certain time interval. For several cases, the optimization is attained at unstable orbits and this leads to the problem of determining sub-optimal good solutions which are stable, and therefore accessible in the system under noise perturbation. We use methods from symbolic dynamics and chaotic control. We discuss further developments for which the system is perturbed by an external system.

**Keywords:** Nonlinear dynamics, Chaotic attractor, Chaotic control, Iteration.

### Acknowledgements

This work was supported by FCT - Fundação para a Ciência e a Tecnologia under the project PEst-OE-MAT-UI0117-2014.

### References

- [1]Joseph D. Skufca, Erik M. Bolt. (2003) Feedback control with finite accuracy: more knowledge and better control for free, *Physica D*, 179, 18-32.
- [2]J.Sousa Ramos, J.P.Lampreia, (1997) Symbolic Dynamics of Bimodal Maps, *Portugaliae Mathematica*, vol 54 fasc. 1, pag 1-18.
- [3]May, Robert M. (1976). Simple mathematical models with very complicated dynamics. *Nature*, 261 (5560): 459–467.



## Some results on the Frobenius coin problem

Manuel B. Branco<sup>1</sup>

<sup>1</sup>Universidade de Évora, Departamento de Matemática e CIMA, Portugal

**Corresponding Author:** *mbb@uevora.pt*

### Abstract

Let  $\mathbb{N}$  denote the set of nonnegative integers. A numerical semigroup is a subset  $S$  of  $\mathbb{N}$  closed under addition, it contains the zero element and has finite complement in  $\mathbb{N}$ . Given a nonempty subset  $A$  of  $\mathbb{N}$  we will denote by  $\langle A \rangle$  the submonoid of  $(\mathbb{N}, +)$  generated by  $A$ , that is,

$$\langle A \rangle = \{ \lambda_1 a_1 + \cdots + \lambda_n a_n \mid n \in \mathbb{N} \setminus \{0\}, a_i \in A, \lambda_i \in \mathbb{N} \text{ for all } i \in \{1, \dots, n\} \}.$$

It is well known (see for example [5]) that  $\langle A \rangle$  is a numerical semigroup if and only if  $\gcd(A) = 1$ .

If  $S$  is a numerical semigroup and  $S = \langle A \rangle$  then we say that  $A$  is a system of generators of  $S$ . Moreover, if  $S \neq \langle X \rangle$  for all  $X \subsetneq A$ , then we say that  $A$  is a minimal system of generators of  $S$ . It is well known that [see [5], Theorem 2.7] every numerical semigroup admits a unique minimal system of generators, which in addition is finite. The cardinality of its minimal system of generators is called the embedding dimension of  $S$ , denoted by  $e(S)$ .

Following a classic line, two invariants have special relevance to a numerical semigroups: the greatest integer that does not belong to  $S$ , called the Frobenius number of  $S$  denoted by  $F(S)$ , and the cardinality of  $\mathbb{N} \setminus S$ , called the genus of  $S$  denoted by  $g(S)$ .

The Frobenius coin problem (often called the linear Diophantine problem of Frobenius) consists in finding a formula, in terms of the elements in a minimal system of generators of  $S$ , for computing  $F(S)$  and  $g(S)$  (for a complete overview see [1]). This problem was solved by Sylvester for numerical semigroups with embedding dimension two. Sylvester demonstrated that if  $\{n_1, n_2\}$  is a minimal system of generators of  $S$ , then  $F(S) = n_1 n_2 - n_1 - n_2$  and  $g(S) = \frac{1}{2}(n_1 - 1)(n_2 - 1)$ . The Frobenius coin problem remains open for numerical semigroups with embedding dimension greater than or equal to three.

In this talk we will present some classes of numerical semigroups for which this problem is solved (see for instance [2], [3], [4], [6], [7] and [8]).

**Keywords:** Numerical semigroup, Frobenius number, embedding dimension, genus.

## Acknowledgements

The researcher belongs to the Centro de Investigação em Matemática e Aplicações, Universidade de Évora, Project UID-MAT-04674-2013, a research centre supported by FCT (Fundação para a Ciência e a Tecnologia, Portugal).

## References

- [1]J. L. Ramirez Alfonsín, The Diophantine Frobenius Problem, Oxford University Press, London (2005).
- [2]J. C. Rosales, M. B. Branco, The Frobenius problem for numerical semigroups with multiplicity four. *Semigroup Forum* 83 (2011), no. 3, 468-478
- [3]J. C. Rosales, M. B. Branco, The Frobenius problem for numerical semigroups. *J. Number Theory* 131 (2011), no. 12, 2310-2319.
- [4]J. C. Rosales, M. B. Branco, Irreducible numerical semigroups. *Pacific J. Math.* 209 (2003), no. 1, 131-143. 20M14.
- [5]J. C. Rosales, P. A. García-Sánchez, Numerical semigroups, *Developments in Mathematics*, vol.20, Springer, New York, (2009).
- [6]J. C. Rosales, M. B. Branco and D. Torrão, The Frobenius problem for Thabit numerical semigroups, *Journal of Number Theory*, 155, 85-99, (2015).
- [7]J. C. Rosales, M. B. Branco and D. Torrão, The Frobenius problem for Repunit numerical semigroups, submitted.
- [8]J. C. Rosales, M. B. Branco and D. Torrão, The Frobenius problem for Mersenne numerical semigroups, *Mathematische Zeitschrift* . Z. DOI 10.1007/s00209-016-1781-z.

## Random sample sizes in one-way ANOVA with fixed effects

Célia Nunes<sup>1</sup>, Gilberto Capistrano<sup>2</sup>, Dário Ferreira<sup>1</sup>, Sandra S. Ferreira<sup>1</sup> and João Tiago Mexia<sup>3</sup>

<sup>1</sup>Department of Mathematics and Center of Mathematics and Applications, University of Beira Interior, Portugal

<sup>2</sup>School of Business and Development of Excellence - ENDEX, Pouso Alegre, Brasil

<sup>3</sup>Center of Mathematics and its Applications, Faculty of Science and Technology, New University of Lisbon, Portugal

**Corresponding Author:** *celian@ubi.pt*

### Abstract

Analysis of variance (ANOVA) is a well known statistical method used in several research areas. The aim of this work is to extend the theory of one-way fixed effects ANOVA to situations where the samples sizes may not be previously known. An illustrative example of this is the collection of observations during a fixed time period in a study comparing, for example, several pathologies of patients arriving at a hospital, see e.g. [1–3]. In these situations it is more appropriate to consider the sample sizes as realizations,  $n_1, \dots, n_m$ , of independent random variables,  $N_1, \dots, N_m$ . In this work we will assume that the samples were generated by Poisson counting processes and we present the test statistics and their conditional and unconditional distributions, under the assumption that we have random sample sizes. We also show how to compute correct critical values, see [3]. The applicability of the proposed approach is illustrated considering a real medical data example. Finally, we carry out with a simulation study, to compare and relate the performance of our approach with the common ANOVA.

**Keywords:** Random sample sizes, one-way fixed effects ANOVA, correct critical values, Poisson distribution, cancer registries.

### Acknowledgements

This work was partially supported by national funds of FCT-Foundation for Science and Technology under UID-MAT-00212-2013 and UID-MAT-00297-2013.

## References

- [1]J.T. Mexia, C. Nunes, D. Ferreira, S.S. Ferreira, E. Moreira (2011) Orthogonal fixed effects ANOVA with random sample sizes, *Proceedings of the 5th International Conference on Applied Mathematics, Simulation, Modelling (ASM'11)*, pp. 84-90.
- [2]C. Nunes, D. Ferreira, S.S. Ferreira, J.T. Mexia (2012)  $F$ -tests with a rare pathology, *J. Appl. Stat.* v. 39, n.3, pp. 551-561.
- [3]C. Nunes, D. Ferreira, S.S. Ferreira, J.T. Mexia (2014) Fixed effects ANOVA: an extension to samples with random size, *J. Stat. Comput. Simulation*,v.84, n.11, pp. 2316-2328.

# Axisymmetric Motion of a Proposed Generalized Non-Newtonian Fluid Model with Shear-dependent Viscoelastic Effects

Fernando Carapau<sup>1</sup>, Paulo Correia<sup>1</sup>, Luís M. Grilo<sup>2</sup> and Ricardo Conceição<sup>3</sup>

<sup>1</sup>Universidade de Évora, Departamento de Matemática e CIMA, Portugal

<sup>2</sup>Instituto Politécnico de Tomar, Unidade Departamental de Matemática e Física e CMAT-FCT-UNL, Portugal

<sup>3</sup>PhD. student from the Renewable Energies Chair, Universidade de Évora, Portugal

**Corresponding Author:** *flc@uevora.pt*

## Abstract

Three-dimensional numerical simulations of non-Newtonian fluid flows are a challenging problem due to the particularities of the involved differential equations leading to a high computational effort in obtaining numerical solutions, which in many relevant situations becomes infeasible. Several models have been developed along the years to simulate the behavior of non-Newtonian fluids together with many different numerical methods. In this work we use a one-dimensional hierarchical approach to a proposed generalized third-grade fluid with shear-dependent viscoelastic effects model. This approach is based on the Cosserat theory related to fluid dynamics and we consider the particular case of flow through a straight and rigid tube with constant circular cross-section. With this approach, we manage to obtain results for the wall shear stress and mean pressure gradient of a real three-dimensional flow by reducing the exact three-dimensional system to an ordinary differential equation. This one-dimensional system is obtained by integrating the linear momentum equation over the constant cross-section of the tube, taking a velocity field approximation provided by the Cosserat theory. From this reduced system, we obtain the unsteady equations for the wall shear stress and mean pressure gradient depending on the volume flow rate, Womersley number, viscoelastic coefficients and the flow index over a finite section of the tube geometry. Attention is focused on some numerical simulations for constant and non-constant mean pressure gradient using a Runge-Kutta method.

**Keywords:** One-dimensional model, generalized third-grade model, shear-thickening fluid, shear-thinning fluid, Cosserat theory.

## Acknowledgements

The researcher belongs to the Centro de Investigação em Matemática e Aplicações, Universidade de Évora, Project UID-MAT-04674-2013, a research centre supported by FCT (Fundação para a Ciência e a Tecnologia, Portugal).

## References

- [1]Carapau, F., Correia, P., Grilo, L.M., and Conceição, R. (2017) Axisymmetric Motion of a Proposed Generalized Non-Newtonian Fluid Model with Shear-dependent Viscoelastic Effects, *IAENG International Journal of Applied Mathematics*, accepted for publication.
- [2]Caulk, D.A., and Naghdi, P.M. (1987) Axisymmetric motion of a viscous fluid inside a slender surface of revolution, *Journal of Applied Mechanics*, v.54, n.1, pp. 190–196.
- [3]Fosdick, R.L., and Rajagopal, K.R. (1980) Thermodynamics and stability of fluids of third grade, *Proc. R. Soc. Lond. A.*, v.339, pp. 351–377.

## Statistical agreement between two methods of measurement

Luís M. Grilo<sup>1,2</sup> and Helena L. Grilo<sup>2</sup>

<sup>1</sup>Instituto Politécnico de Tomar, Departamento de Matemática e Física, and Centro de Matemática e Aplicações (CMA), FCT/UNL, Portugal

<sup>2</sup>Centro de Sondagens e Estudos Estatísticos, Instituto Politécnico de Tomar, Portugal

**Corresponding Author:** *lgrilo@ipt.pt*

### Abstract

To measure a variable in a continuous scale it is possible, in some situations, to use more than one measurement method, as it happens in the fields of Health and Engineering. However, it is impossible, at least nowadays, to eliminate completely the error associated to those methods. When there are two alternative measurement methods, a new versus an established method, considered sometimes the “gold standard”, it is necessary to analyse statistically the agreement between them, before using both interchangeably or replacing the old method by the new one [1-10]. Based on actual clinical data, the attractive (non)parametric approaches based on the well known limits of agreement are applied and the assumptions of each one is discussed (the limits of agreement are estimated after a Box Cox transformation, for a particular linear regression and a bootstrap resampling method is also used, in order to obtain robust confidence intervals for the mean and median of differences, which estimate the bias). The conclusions, in this particular case study, point out to a statistically significant agreement between both methods.

**Keywords:** Bootstrap, Box-Cox transformation, Clinical data, Limits of agreements, Nonparametric.

### Acknowledgements

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID-MAT-00297-2013 (Centro de Matemática e Aplicações).

### References

- [1] Barnhart, H. X., Haber, M. J. and Lin, L. I. (2007). An overview on assessing agreement with continuous measurements. *Journal of Biopharmaceutical Statistics*, 17(4), 529-69.

- [2]Bland, J. and Altman, D. (1999). Measuring agreement in method comparison Studies. *Statistical Methods in Medical Research*, 8(2), 135-160.
- [3]Bland, J. and Altman, D. (1986). Statistical methods for assessing agreement between two methods of clinical measurement. *The Lancet*, 307-10.
- [4]Carpenter, J. and Bithell, J. (2000). Bootstrap confidence intervals: when, which, what? A practical guide for medical statisticians, *Statistics in Medicine*; 19:1141-64.
- [5]Costa-Santos, C., Antunes, L., Souto, A. and Bernardes, J. (2010). Assessment of disagreement: a new information-based approach. *Ann Epidemiol.*, 20 (7), 555-61.
- [6]Grilo, L. M. and Grilo, H. L. (2016). Robust statistical approaches to assess the degree of agreement of clinical data. *ICNAAM 2015, AIP Conf. Proc.* 1738, 470005.
- [7]Grilo, L. M., Henriques, R. S., Correia, P. C. and Grilo, H. L. (2015b). Statistical analysis of psychomotor therapy in children with attention-deficit/hyperactivity disorder. *WSEAS Transactions on Biology and Biomedicine*, ISSN/E-ISSN: 1109-9518 / 2224-2902, Vol. 12, 6, 39-43.
- [8]Grilo, L. M., Henriques, R. S., Correia, P. C. and Grilo, H. L. (2015a). Attention deficit/hyperactivity disorder in children. A statistical approach. *ICNAAM 2014, AIP Conf. Proc.* 1648, 840007-1–840007-4.
- [9]Grilo, L. M. and Grilo, H. L. (2012). Comparison of clinical data based on limits of agreement. *Biometrical Letters*, 49, 1, 45-56.
- [10]Yellareddygar S. K. R. and Gudmestad N. C. (2017). Bland-Altman comparison of two methods for assessing severity of Verticillium wilt of potato. *Crop Protection (Elsevier)* 101, 68-75.



## Exploring R features for Experimental Designs

Teresa A. Oliveira<sup>1,2</sup> and Amílcar Oliveira<sup>1,2</sup>

<sup>1</sup>Universidade Aberta, Portugal

<sup>2</sup>Centro de Estatística e Aplicações (CEAUL), Portugal

**Corresponding Author:** *teresa.oliveira@uab.pt*

### Abstract

The ability to obtain and filter information from data, distinguishes the best professionals in several areas and is pointed out as one of the most desirable skills in the current competitive World. Now-a-days, it is then crucial for statisticians and researchers to possess competences on data manipulation, data analysis and data visualization. The role of software R is very well known as the currently preferred one for such issues. R is a powerful programming language for loading, manipulating, transforming, and visualizing data. Besides that, R is an environment for statistical computing and graphics that has become increasingly popular in academic and research activities, thanks to its extensibility in conjunction with the efforts of a highly active open source community. Some of the classical and advanced methodologies of Experimental Design considering a Single Factor will be illustrated, exploring the R features with real and with simulated data, aiming to foster research and further international collaborations in these areas.

**Keywords:** Experimental Design, ANOVA one-way layout, Block Designs, Balanced Incomplete Block Designs, Latin Squares, Graeco-Latin Squares, R software.

### Acknowledgements

This research was partially sponsored by national funds through the Fundação Nacional para a Ciência e Tecnologia, Portugal - FCT under the project PEst-OE-MAT-UI0006-2013.

### References

- [1]Montgomery, D. C. (2012). Design and analysis of experiments (8th ed.). New York, NY: Wiley.
- [2]Gromping U (2011a). CRAN Task View: Design of Experiments (DoE) Analysis of experimental Data. Version 2011-08-10, <http://CRAN.R-project.org/view=ExperimentalDesign>.
- [3]<http://cran.r-project.org/doc/manuals/R-intro.html>.

## Numerical Schemes to Solve Some MTFDE's

M. Filomena Teodoro<sup>1,2</sup>

<sup>1</sup>CINAV, Naval Academy, Portuguese Navy and CEMAT, IST, Lisbon Univ., Portugal

**Corresponding Author:** *maria.alves.teodoro@marinha.pt*

### Abstract

Backward-forward equations, equations a differential equation with delayed and advanced arguments, appears in a wide number of applied sciences such as biology, physics, economy, control, acoustics, etc. We achieve in literature mathematical several models which are included in this family of functional differential equations, for example see [1,3–5,19,2,13,14]. In some cases [6,7], the methods to solve delay differential equations can be adapted to solve mixed type functional differential equations (MTFDE's). Some schemes using the method of steps, B-splines, collocation, finite differences and finite element method were developed to solve linear, autonomous and non-autonomous MTFDE's [15,8,9]. Relatively to some non-linear advanced-retarded equations from nervous conduction [4] and human phonation [19,12] have been numerically solved using an adapted method of steps, Newton method an extended version of the numerical schemes applied to linear case [10,11,16]. Recently, in [18], a preliminary of a non-linear MTFDE with symmetric delay and advance using collocation and radial basis functions was done. The numerical results using collocation, B-splines and exponential radial functions are similar, but it is necessary to perform more simulations using different basis of radial functions. An overview about the numerical schemes to solve some MTFDE's will be given.

**Keywords:** Mixed type functional differential equation, numerical approach, numerical solution, collocation, finite differences, finite element method, B-splines, radial basis function.

### Acknowledgements

This work was supported by Portuguese funds through the CINAV, Portuguese Naval Academy, and FCT, through the CEMAT, University of Lisbon, project UID-Multi-04621-2013.

## References

- [1]K.A. Abell et al. (2005) Computation of mixed type functional differential boundary value problems, *Siam Journal on Applied Dynamical Systems*, v.4, n.3, pp.755–781.
- [2]Alvarez-Rodriguez, U. et al. (2017) Advanced-retarded differential equations in quantum photonic systems, *Scientific Reports*, v.7, art. 42933.
- [3]J. Bell (1984) Behaviour of some models of myelinated axons, *IMA J. mathematics applied in medicine and biology*, v.1, pp.149–167.
- [4]H. Chi, J. Bell, B. Hassard (1986) Numerical solution of a nonlinear advance-delay-differential equation from nerve conduction, *J. Mathematical Biology*, v.24, pp.583–601.
- [5]K. Ishizaka, M. Matsudaira (1972) Fluid Mechanical Considerations of Vocal Cord Vibration, *Speech Comm. Res. Lab. CA*. Mon 8.
- [6]V. Iakovleva and C. Vanegas (2005) On the Solution of differential equations with the delayed and advanced arguments, *Elect. J. Differential Equation*, Conference 13, pp.57–63.
- [7]N.J. Ford and P.M. Lumb (2009) Mixed-type functional differential equations: a numerical approach, *J. Computational and Applied Mathematics*, v.229, n.2, pp.471–479.
- [8]Lima, P.M. et al. (2010) Analytical and Numerical Investigation of Mixed Type Functional Differential Equations, *J. Comp. and Applied Mathematics*, v.234, n.9, pp.2732–2744.
- [9]Lima, P.M. et al. (2010) Finite Element Solution of a Linear Mixed-Type Functional Differential Equation, *Numerical Algorithms*, v.55, pp.301–320.
- [10]Lima, P.M. et al. , in: S. Pinelas et al. (Ed.) Analysis and Computational Approximation of a Forward-Backward Equation Arising in Nerve Conduction, *Differential and Difference Equations with Applications*, (Springer Proc. Math & Stat, NY 2013), n.47, pp.475–483.
- [11]Lima, P.M. et al. (2014) Computational Methods for a mathematical model of propagation of nerve impulses in myelinated axons, *App. Num. Math.*, v.85, pp.38.
- [12]J. C. Lucero (2008) Advanced-Delay Equations for Aerolastics Oscillations in Physiology, *Biophys Rev. Letters*, v.3, n.1, pp.125–133.
- [13]Pontryagin, L.S. et al., *The mathematical Theory of Optimal Process*, (Intersc., NY, 1962)
- [14]Rustichini, A. (1989) Functional differential equations of mixed type: The linear autonomous case, *Journal of Dynamics and Differential Equations*, v.1, n.2, pp.121.
- [15]Teodoro, M.F. et al. (2009) New approach to the numerical solution of forward-backward equations, *Frontiers of Mathematics on China*, v.4, n.1, pp.155–168.
- [16]Teodoro, M.F. (2017) Numerical Solution of a Delay-Advanced Equation from Physiology, *Applied Mathematics & Information Sciences*, v.11, n.5, pp.1287–1297.
- [17]Teodoro, M.F. (2017) Numerical Solution of a Delay-Advanced Equation from Acoustics, *International Journal of Mechanics*, v.11, pp.107–114.
- [18]Teodoro, M.F. (2017) Approximating a Retarded-Advanced Differential Equation Using Radial Basis Functions, In Gervasi, O. et al. (Eds.), Computational Science and Its Applications ICCSA 2017, *Lecture Notes in Computer Science*, v.10408(V), pp.33–43.
- [19]Titze, I.R., *Principles of Voice Production*, (Prentice-Hall, Englewood Cliffs, 1994)

**Contributed Talk**



# Anova Analysis and Related Techniques for Structured Families of Symmetric Stochastic Matrices

Cristina Dias<sup>1,3</sup>, João Miranda<sup>1,5</sup>, Carla Santos<sup>2,3</sup> and João Tiago Mexia<sup>3,4</sup>

<sup>1</sup>Instituto Politécnico de Portalegre, Departamento de Tecnologias, Portugal

<sup>2</sup>Departamento de Matemática e Ciências Físicas do Instituto Politécnico de Beja, Portugal

<sup>3</sup>Centro de Matemática e Aplicações da Universidade Nova de Lisboa (CMA), Portugal

<sup>4</sup>Departamento de Matemática da Universidade Nova de Lisboa, Portugal

<sup>5</sup>Centro de Recursos Naturais e Ambiente Instituto Superior Técnico (CERENA), Portugal

**Corresponding Author:** *cpsd@estgp.pt*

## Abstract

Symmetric stochastic matrices with a dominant eigenvalue  $\gamma$  and the corresponding eigenvector  $\alpha$  appears in many applications. Such matrices can be written as  $M = \mu + \bar{E} = \gamma\alpha\alpha^t + \bar{E}$ . Thus  $\beta = \gamma\alpha$  will be the structure vector. When the matrices in such families correspond to the treatments of a base design we can carry out a ANOVA like analysis of the action of the treatments in the model on the structured vectors. This analysis can be transversal and longitudinal. In the latter we work with vectors contrasts in the components of the structure vector, while in the former we work with the homologous components of that vector. The analysis will be briefly considered at the end of our presentation.

**Keywords:** Anova Analysis, Structured Families, Transversal and Longitudinal Analysis.

## Acknowledgements

Research partially funded by FCT Fundação para a Ciência e a Tecnologia, Portugal, through the projects PEst-OE-MAT-UI0006-2014, UID-MAT-00006-2013.

## References

- [1]Areia A. (2009). Séries emparelhadas de estudos. Ph.D. Thesis. Évora University.
- [2]P. K. Ito (1980). Robustness of Anova and Macanova Test Procedures, P. R. Krishnaiah (ed) Handbook of Statistics1, Amsterdam: North Holland, 1, 199-236.

- 
- [3]J. T. Mexia (2009). Standardized Orthogonal Matrices and Decomposition of the Sum of Squares for Treatments. *Trabalhos de Investigação*, nº2, Departamento de Matemática, Faculdade de Ciências e Tecnologia da Universidade Nova de Lisboa.
  - [4]C. Dias (2013). Modelos e Famílias de Modelos para Matrizes Estocásticas Simétricas. Ph.D. Thesis, Évora University.
  - [5]M. M. Oliveira and J. T. Mexia (1999). F Tests for Hypothesis on the Structure Vectors of Series. *Discussiones Mathematicae*, 19(2), 345-353.
  - [6]H. Scheffé (1959). *The Analysis of Variance*, New York: John Wiley Sons.

## Regional Frequency Analysis of Extreme Climate Phenomena

J.F. Santos<sup>1</sup>, M. M. Portela<sup>2</sup> and I. Pulido-Calvo<sup>3</sup>

<sup>1</sup>Instituto Politécnico de Beja, Portugal

<sup>2</sup>Instituto Superior Técnico, Departamento de Eng. Civil, Portugal

<sup>3</sup>Universidad de Huelva, Departamento de Ciencias Agroforestales, Espanha

**Corresponding Author:** *joaof.santos@ipbeja.pt*

### Abstract

This study investigated the frequency of droughts for the period September 1910 to October 2004 in mainland Portugal, based on monthly precipitation data from 144 raingauges distributed across the country. The drought events were characterized using the standardized precipitation index (SPI) [5] applied to timescales of 1, 3, 6 and 12 consecutive months [4]. Based on the SPI time scale series a regional frequency analysis [1], [2] of drought magnitudes was undertaken using two approaches: annual maximum series (AMS) and partial duration series (PDS), [6]. Three spatially defined regions (north, central and south) were identified by cluster analysis and analyzed for homogeneity [3]. Maps of drought magnitude were developed using a kriging technique for several return periods. Similar uniform spatial patterns were found throughout the country using the AMS and PDS approaches. For several SPI timescales, a comparison of the observed and estimated maximum magnitude (269- year empirical return period) showed that the AMS combined with the selected probability distribution models (Pearson type III, general Pareto and Kappa) provided better results than the PDS approach combined with the same models. A general and simplified characterization of drought duration revealed a relatively uniform pattern of droughts events across the country.

**Keywords:** Standardized precipitation index (SPI), Annual maximum series, Partial duration series.

### Acknowledgements

### References

- [1] Hosking, J.R.M. and J.R. Wallis (1993), Some statistics useful in regional frequency analysis, WRR, 29(1), 271-281.



- [2]Hosking, J.R.M. and J.R. Wallis (1997), Regional frequency analysis – An approach based on L-moments, 224 pp., Cambridge University Press, UK.
- [3]Santos, J.F., I. Pulido-Calvo and M.M. Portela (2010), Spatial and temporal variability of droughts in Portugal, WRR, 46, W03503.
- [4]Agnew, C.T. (2000), Using the SPI to identify drought, Drought Network News, 12, 6-12.
- [5]McKee, T. B., N. J. Doesken, and J. Kleist (1993), The relationship of drought frequency and duration to time scales, in Proceedings of the 8th Conf. on Ap. Clim., Ame. Met. Soc., 179-184.
- [6]Haan, C.T. (Ed.) (1977), Statistical Methods in Hydrology, 378 pp., The Iowa State University Press, Iowa, USA.

## Oscillators on a Cantor set

Luís Bandeira<sup>1,2</sup> and Carlos C. Ramos<sup>1,2</sup>

<sup>1</sup>Universidade de Évora, Departamento de Matemática, Portugal

<sup>2</sup>Centro de Investigação em Matemática e Aplicações (CIMA), Portugal

**Corresponding Author:** *lmzb@uevora.pt, ccr@uevora.pt*

### Abstract

We study a chain of linear oscillators which change in time with a varying number of degrees of freedom. The dynamical evolution is determined by a growing model and we analyze the behavior of the system and its dependence on the parameters such as mass and coupling strength. We obtain recursive rules for the eigenvalues and eigenfunctions which allows the determination of the exact solutions. Since dimension is varying is necessary to use a suitable formalism: we use Fock space formalism, operators similar to those used in quantum field theory and certain representations of  $C^*$ -algebras. The method can be made general, nevertheless we present details on the case the chain growing model is determined by a Cantor set recursion type. This model may be suitable for studying vibrations on non-homogeneous materials.

**Keywords:** Linear oscillators, Cantor set, Spectrum, Iteration.

### Acknowledgements

This work was supported by FCT - Fundação para a Ciência e a Tecnologia under the project PEst-OE-MAT-UI0117-2014.

### References

- [1]Bandeira, L., Correia Ramos, C. (2016) Transition matrices characterizing a certain totally discontinuous map of the interval, *Journal of Mathematical Analysis and Applications*, Volume 444, Issue 2, 1274-1303.
- [2]Bandeira, L., Correia Ramos, C. (2015) On the spectra of certain matrices and the iteration of quadratic maps, *SeMA J.* 67, 51-69.
- [3]Correia Ramos, C., Martins, N., and Pinto, P.R. (2017) Toeplitz algebras arising from escape points of interval maps, *Banach J. Math. Anal.* Volume 11, Number 3 , 536-553.
- [4]Correia Ramos, C., Martins, N., and Pinto, P.R. (2013) On  $C^*$ -algebras from interval maps, *Complex Analysis and Operator Theory*, Vol.7, 221-235.

# Whos not afraid of Big Data?

João A. Branco<sup>1,2</sup> and Ana M. Pires<sup>1,2</sup>

<sup>1</sup>Instituto Superior Técnico, Universidade de Lisboa, Portugal

<sup>2</sup>Center for Computational and Stochastic Mathematics (CEMAT), Portugal

**Corresponding Author:** *jbranco@math.ist.utl.pt*

## Abstract

The recent massive production of high-dimensional data has brought great difficulties and concomitant challenges to statistics [1] since its usual methods were not designed to cope with such kind of data. High dimensionality triggers the curse of dimensionality and unexpected behaviour of some statistical tools may surprise even those aware of the intricacies of multidimensional spaces with a large number of dimensions. We look at the Mahalanobis distance [2], a tool that is crucial to the functioning of the traditional multivariate statistical methods, and see how it progresses as  $p$  approaches  $n$  and when it is greater than  $n$ . Can the Mahalanobis distance keep the fundamental role in high-dimensional spaces as it does in low dimensional spaces ( $p \ll n$ )? And if it does not what are the consequences? We will attempt to answer these questions and discuss the serious practical implications of the theoretical results accomplished in this research.

**Keywords:** Curse of Dimensionality, High dimensional data, Mahalanobis distance.

## Acknowledgements

## References

- [1]Johnstone, I.M. and Titterington, D.M. (2009). Statistical challenges of high-dimensional data. Phil. Trans. R. Soc. A 367, 4237-4253.
- [2]Mahalanobis, P.C. (1936). On the generalized distance in Statistics. Proceedings of the National Institute of Science of India 2, 49-55.

## Risk behaviour in the asset management industry

Cristina Dias<sup>1,2</sup>, João Romacho<sup>1,3</sup> and Maria José Varadinov<sup>1,3</sup>

<sup>1</sup>Polytechnic Institute of Portalegre, Portugal

<sup>2</sup>CMA– Center for Mathematics and Applications of the Nova University of Lisbon,  
Portugal

<sup>3</sup>Interdisciplinary Coordination for Research and Innovation (C3i), Portalegre, Portugal

**Corresponding Author:** *jromacho@estgp.pt*

### Abstract

This study analyses the risk behaviour among the mutual funds of several countries. To achieve this aim, it is used the Brown, Harlow and Starks (1996) methodology applied in different settings. Thus, the risk behaviour is analysed in subperiods with the same duration and in subperiods that correspond to different market cycles, the characteristics of the funds are also contemplated (the dimension of their portfolio and its period of activity), as well as the possible effect of the survivorship bias. The outcomes obtained show the existence of strategic behaviour among the mutual funds from the European Union, this is stronger among the funds from Belgium, Spain and United Kingdom. Furthermore, with the exception of the United Kingdom, this behaviour is stronger among the funds with a smaller period of activity and in the most recent period. So, it seems that, on the one hand, the greatest strategic interaction among funds with smaller period of activity can occur from its greatest audacity, unlike the most experienced funds that tend to reveal higher caution. On the other hand, the growth of the markets from the European Union concerning the number of funds seems to contribute to the increase of the strategic behaviour.

**Keywords:** Risk behaviour, Competition, Strategic behavior, Mutual funds.

### Acknowledgements

### References

- [1]Brown, K., Harlow, W. Starks, L. (1996). On tournament and temptations: an analysis of managerial incentives in the mutual fund industry. *Journal of Finance*, 51(1), 85-110.
- [2]Busse, J. (2001). Another look at mutual fund tournaments. *Journal of Financial and Quantitative Analysis*, 36(1), 53-73

- [3]Elton, E., Gruber, M., Blake, C., Krasny, Y. Ozelge, S. (2010). The effect of holdings data frequency on conclusions about mutual fund behavior. *Journal of Banking Finance*, 34(5), 912-922.
- [4]Ramos, S. (2009). The size and structure of the world mutual fund industry. *European Financial Management*, 15(1), 145-180.
- [5]Qiu, J. (2003). Termination risk, multiple managers and mutual fund tournaments. *European Finance Review*, 7(2), 161-190.
- [6]Schwarz, C. (2008). Mutual fund tournaments: the sorting bias and new evidence. Working Paper, University of California at Irvine.

## The Stern-Brocot tree

Isabel Colaço<sup>1</sup>

<sup>1</sup>Instituto Politécnico de Beja, Portugal

**Corresponding Author:** *isabel.colaco@ipbeja.pt*

### Abstract

Rational number system is the core of modern mathematics. At least for the computational point of view we all aim that our objects and methods are rational. There is an (extended) infinite binary tree, called the Stern-Brocot tree, that provides a list of all positive rational numbers in reduced forms. This is not a new object, it was discovered independently by a German mathematician and a French clockmaker around 1860 [1], but it seems that this remarkable combinatorial object is rather unknown. We can inductively construct a node of the Stern-Brocot tree by taking the median of its first Left and Right ancestors and so the Farey series emerges from the left subtree. After a controlled manipulation of this iteration formula we realise two other clarifying models of the Stern-Brocot tree. The first describes a positive rational as an element of the free semigroup generated by two unimodular 2 by 2 matrices L and R. Multiplication by one of this matrices, L or R, translates the median operation and keeps track of the relevant adjancency. We get in this way a one-to-one representation of positive rationals by finite binary sequences and have a natural notion of an irrationality. It is also clear how to describe the best rational approximations to an irrational within an interval, method that can be illustrated starting from the Eulers description of  $e$  as an infinite continued fraction [2]. The other manifestation of the Stern-Brocot tree is geometric in nature, where a positive rational number is a point of the first quadrant of the integer plane lattice that is a visible point from the origin (0,0) and the median operation is just addition in the lattice. This relates to the projective extension of the rational affine line and thus to the circle.

**Keywords:** Median, Farey series, Euclidean algorithm, Continued fraction, Best rational approximations, Visible points, Projective rational line, Rational parametrization of a conic.

### Acknowledgements

## References

- [1] Ronald L. Graham, Donald E. Knuth, Oren Patashnik. Concrete Mathematics, Reading, Massachussets: Addison-Wesley, 1994.
- [2] Leonhard Euler, *Introductio in Analysin Infinitorum*. Tomus primus, Lausanne, 1748.

## Improving the Extremal Index Blocks Estimator

Dora Prata Gomes<sup>1</sup> and Manuela Neves<sup>2</sup>

<sup>1</sup>Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, and Centro de Matemática e Aplicações, Portugal

<sup>2</sup>Instituto Superior de Agronomia, Universidade de Lisboa and Centro de Estatística e Aplicações, Portugal

**Corresponding Author:** *dsrp@fct.unl.pt*

### Abstract

The main objective of statistics of extremes is the estimation of parameters of rare events. One of these parameters is the extremal index,  $\theta$ , that measures the degree of local dependence in the extremes of a stationary process. Clusters of extreme values are linked with incidences and durations of catastrophic phenomena, an important issue in areas like environment, finance, insurance among others. The extremal index is a key parameter and its estimation has been greatly addressed in literature. Here we focus on the estimation of  $\theta$  using the blocks method, which identifies clusters and constructs estimates based on these clusters. This so-called blocks estimator of  $\theta$  has been intensively studied in the literature. Hsing [1] and Weissman and Novak [2] proved its consistency and its asymptotic normality under suitable mixing conditions. However there are two parameters, which determine the clusters and consequently the blocks estimates of  $\theta$ : a threshold and a cluster identification scheme parameter, the block length. We have conducted a simulation study where we have applied a procedure given in Canto e Castro [3] based on the definition of clusters of exceedances through an adequate threshold choice in each block. Comparison with other procedures results is shown. An application to daily mean flow discharge rate data, in the hydro-metric station of Fragas da Torre in Paiva River, collected from 1 October 1946 to 30 September 2006 is performed.

**Keywords:** Blocks estimators, clusters of extreme values, daily mean flow discharge rate, extremal index, threshold choice.

### Acknowledgements

Research partially funded by FCT Fundação para a Ciência e a Tecnologia, Portugal, through the projects UID-MAT-00297-2013 (CMA) and UID-MAT-00006-2013 (CEAUL).



## References

- [1]Hsing, T. (1993). Extremal index estimation for a weakly dependent stationary sequence. *Ann. Statist.* 21, 2043-2071.
- [2]Weissman, I. and Novak, S. Yu. (1998). On blocks and runs estimators of the extremal index. *J. Statist. Plann. Inference* 66, 281-288.
- [3]Canto e Castro, L. (1992). Estudo de um Método de Estimação do Índice Extremal. I Congreso Ibero-Americano de Estadística e Investigación Operativa, Cáceres, 157-201.

# A comparison of the performance of restoration-classification models with spatial data

Luís Filipe Domingues<sup>1</sup> and José G. Dias<sup>2</sup>

<sup>1</sup>Escola Superior de Tecnologia e Gestão, Instituto Politécnico de Beja, Portugal

<sup>2</sup>Instituto Universitário de Lisboa (ISCTE-IUL), BRU-IUL, Portugal

**Corresponding Author:** *luis.domingues@ipbeja.pt*

## Abstract

This paper aims to compare the performance of restoration-classification models in the context of spatial data under an increasing overlap of the distributions of the segments. We focus our attention on the mean field algorithm [3] and the iterative conditional modes algorithm [4]. These algorithms use mean field approximations [1] to tackle the intractable complete likelihood of these hidden Markov models with parameter estimation via the Expectation-Maximization (EM) algorithm [2]. Synthetic Gaussian data spatial structures are used to compare the performance of the models.

**Keywords:** Finite mixture models; Hidden Markov random fields; Mean field theory; Spatial data; Unsupervised segmentation.

## Acknowledgements

## References

- [1]Zhang Y.Y., Brady M. and Smith S. (2001). Segmentation of brain MR images through a hidden Markov random field model and the expectation-maximization algorithm. *IEEE Transactions on Medical Imaging*, 20,45-57.
- [2]Dempster, A. P., Laird, N. M. Rubin, D. B. (1977). Maximum likelihood from incomplete data via EM algorithm. *Journal of the Royal Statistical Society Series B-Methodological*, 39, 1-38
- [3]Celeux, G. et al (2003) EM Procedures Using Mean Field-Like Approximations for Markov Model-Based Image Segmentation. *Pattern Recognition*, 36, 131-144.
- [4]Besag, J. (1986) On the Statistical Analysis of Dirty Pictures. *Journal of the Royal Statistical Society, Series B-Methodological*, 3(48) , 259-302.

## The Modeling of a simple PV and Cooling PV panel using numerical methods

Assia Sellami<sup>1,2,4</sup>, Oumaima Mesbahi<sup>1,2,4</sup>, Khalid kandoussi<sup>1,2</sup>, Rabie El Otmani<sup>1,2</sup>, Abdeloawahed Hajjaji<sup>1,2</sup> and Mouhaydine Tlemçani<sup>3,4</sup>

<sup>1</sup>Université Chouaib Doukkali, Morocco

<sup>2</sup>Ecole Nationale des sciences Appliquées d'EL JADIDA, Morocco

<sup>3</sup>Universidade de Évora, Portugal

<sup>4</sup>ICT, Portugal

**Corresponding Author:** *sellami.assia@gmail.com*

### Abstract

The characteristic of photovoltaic panels (PV) depends on many parameters as irradiation, environmental and internal temperature, etc. . . The modeling of a simple and a cooled PV panel with phase change material [1-2] using different approaches as Lagrange and Nelder-Mead numerical methods [3-4] is presented. A numerical model based on a coupling between Navier-Stokes and general heat transfer equations is simulated and implemented using Matlab and Comsol software. The numerical results show the effect of the main parameters and the cooling system on the PV behavior.

**Keywords:** Photovoltaic system, heat transfer, Lagrange, Nelder-Mead and Navier- Stokes.

### Acknowledgements

The Researchers thanks Instituto de Ciências de Terra, Évora, Portugal for their support.

### References

- [1]A. SELLAMI, R. EL OTMANI, K. KANDOSSI, M. ELJOUAD and A. HAJJAJI (2016). Temperature regulation of PV cell under PCM cooling system. 2016 International Renewable and Sustainable Energy Conference, Proceeding of IEEE Journal, pp.580-584.
- [2]Z. Zhang, X. Fang (2006). Study on paraffin/expanded graphite composite phase change material energy storage material. Energy Conversion and Management, Elsevier.
- [3]C.T. KELLEY (1999). Detection and remediation of stagnation in the nelder-meade algorithm using a sufficient decrease condition. Society for Industrial and Applied Mathematics. Vol. 10, No. 1, pp. 43–55.
- [4]J. Buchmann, M. lüntgen and M. Pohst (1994). A Practical Version of the Generalized Lagrange Algorithm, Experimental Mathematics, 3:3, 199-207.

# Satisfaction with the Tourist Experience: An Applications of the Nonlinear Estimation Model

Pedro M. Cravo<sup>1</sup>

<sup>1</sup>Escola Superior de Tecnologia e Gestão, Instituto Politécnico de Beja, Portugal

**Corresponding Author:** *pedro.cravo@ipbeja.pt*

## Abstract

This paper presents a reflection on the satisfaction about the tourist experience and the evaluation of its determinants. Some authors state that satisfaction doesn't guarantee loyalty [1] [2] [3], although other studies show that there is a deeper intention to return or recommend a destination in the case of satisfied tourists [4] [5]. This is why it's important to understand what determines the satisfaction with the tourist experience. As a case study I tried to verify the situation of tourists that visit the Arade municipalities, in Algarve, Portugal, using the nonlinear estimation model. For that, I analysed part of the results from a study carried out by CIDER, an organization based in the University of Algarve. This study aimed to evaluate the degree of satisfaction of tourists with their experiences in these municipalities. The results obtained, although confirming the importance of satisfaction with the tourist experiences, present some surprises.

**Keywords:** Consumer behaviour, Loyalty, Motivation, Nonlinear estimation, Satisfaction, Tourist experience.

## Acknowledgements

The author wishes to thank CIDER and the University of Algarve for the data provided for this study.

## References

- [1]Yoon, Y. e Uysal, M. (2005). An examination of the effects of motivation and satisfaction on destination loyalty: a structural model. *Tourism Management*, 26, 45-56.
- [2]Ross, R.L. e Iso-Ahola, S.E. (1991). Sightseeing tourists' motivation and satisfaction. *Annals of Tourism Research*, 18(2), 226-237.
- [3]R.L. Oliver. (1999) "Whence Consumer Loyalty?". *Journal of Marketing*, 63, 33-44.
- [4]Bramwell, B. (1998). User satisfaction and product development in urban tourism. *Tourism Management*, 19(1), 35-47.
- [5]Gallarza, M.G. and Saura, I.G. (2006). Value dimensions, perceived value, satisfaction and loyalty: an investigation of university students' travel behaviour. *Tourism Management*, 27, 437-452.

## Comparison of fishing policies for populations with weak Allee effects in a random environment

Nuno M. Brites<sup>1,2</sup> and Carlos A. Braumann<sup>1,2</sup>

<sup>1</sup>Universidade de Évora, Departamento de Matemática, Portugal

<sup>2</sup>Centro de Investigação em Matemática e Aplicações (CIMA), Portugal

**Corresponding Author:** *brites@uevora.pt, braumann@uevora.pt*

### Abstract

In a random environment, we describe the growth of a population subjected to harvesting through stochastic differential equations (as in [1] and [3]). We assume that the population is under the influence of weak Allee effects, that is, at very low values of population size, we observe lower *per capita* growth rates instead of the higher rates one would expect considering the higher availability of resources per individual (see, for example, [5]). The presence of weak Allee effects when population size is low may be due to the difficulty in finding mating partners or in constructing a strong enough group defence against predators. We consider the population natural growth to follow a logistic-like model with Allee effects and that the rate of harvesting is proportional to the existing population and to the effort exerted in the capture. The main goal of this work is to compare the performance of two fishing policies: one with variable effort, here named optimal policy, and the other with constant effort, denoted by sustainable optimal policy. The first allows the fishing effort to vary rapidly and abruptly depending on population size which, in a random environment, also varies constantly. This type of policy is inapplicable from the practical point of view. In addition, this policy requires the estimation of population size at each time instant, which is usually an expensive, inaccurate, and time-consuming task. The second policy considers the application of a constant effort over time and predicts the sustainability of the population as well as the existence of a stationary density for its size (see [2]). This policy has the advantage of being applicable, easily implemented and does not require knowledge of population size at any given time. The performance of the two policies will be assessed by the profit obtained over a finite time horizon. A similar study has been done for the logistic model without Allee effects (see [4]). Using realistic data based on a fish population, we will quantify the reduction in profit when choosing the optimal sustainable policy with constant effort instead of the optimal and inapplicable policy with variable effort.

**Keywords:** Allee effects, constant effort, harvesting policies, profit optimization, stochastic differential equations.

### Acknowledgements

The authors belong to the Centro de Investigação em Matemática e Aplicações, Universidade de Évora, a research centre supported by FCT (Fundação para a Ciência e a Tecnologia, Portugal, ref. UID-MAT-04674-2013). The first author holds a PhD grant from FCT (ref. SFRH-BD-85096-2012).

### References

- [1]Beddington, J.R. and May, R. M. (1977) Harvesting natural populations in a randomly fluctuating environment. *Science*, v.197, pp. 463–465.
- [2]Braumann, C.A. (2002) Variable effort harvesting in random environments: generalization to density-dependent noise intensities. *Math. Biosciences*, v.177 & 178, pp. 229–245.
- [3]Braumann, C.A. (1985) Stochastic differential equation models of fisheries in an uncertain world: extinction probabilities, optimal fishing effort, and parameter estimation. In *Mathematics in Biology and Medicine* (V. Capasso, E. Grosso, and S. L. Paveri-Fontana, editors), Springer, Berlin, pp. 201–206.
- [4]Brites, N.M. and Braumann, C.A. (2017) Fisheries management in random environments: Comparison of harvesting policies for the logistic model. *Fisheries Research*, v.195, pp. 238–246.
- [5]Carlos, C. and Braumann, C.A. (2017) General population growth models with Allee effects in a random environment. *Ecological Complexity*, v.30, pp. 26–33.

## Coupling of numerical algorithms: An application to a nonlinear engineering model

Oumaima Mesbahi<sup>1,2,4</sup>, Assia Sellami<sup>1,2,4</sup>, Khalid kandoussi<sup>1,2</sup>, Rabie El Otmani<sup>1,2</sup>, Abdeloawahed Hajjaji<sup>1,2</sup> and Mouhaydine Tlemçani<sup>3,4</sup>

<sup>1</sup>Université Chouaib Doukkali, Morocco

<sup>2</sup>Ecole Nationale des sciences Appliquées d'EL JADIDA, Morocco

<sup>3</sup>Universidade de Évora, Portugal

<sup>4</sup>ICT, Portugal

**Corresponding Author:** *oumaimames@gmail.com*

### Abstract

Mostely, the precise behavior of a photovoltaic (PV) panel is described by nonlinear analytical equations [1]. In order to solve such equations, this work presents two approximated models of PV panel which will allow the prediction of its characteristics. The models are based on coupling two different numerical methods. The first step is estimating the five parameters of the PV model which is done by using heuristic searching algorithm based on Nelder-Mead method [2-3]. The second step consists on solving the nonlinear equation representing the PV model, by applying the Newton-Raphson iterative method for the first approach and Lagrange polynomial method [4] for the other one. After implementing these different techniques on Matlab, the results are analyzed in order to evaluate the convergence's level and compare the algorithm's consistency and stability of each PV model.

**Keywords:** Photovoltaic panel, Simulation, Lagrange poynimial method, Newton-Raphson method, Nelder-Mead.

### Acknowledgements

The Researchers thanks Instituto de Ciências de Terra, Évora, Portugal for their support.

### References

- [1]M. R. Rashel, A. Albino, T. C. F. Goncalves and M. Tlemçani, Sensitivity Analysis of Environmental and Internal Parameters of a Photovoltaic Cell, in Energy for Sustainability International Conference 2017. Designing Cities Communities for the Future, Funchal, Madeira, 2017.
- [2]J. A. R. M. Nelder, A simplex method for function minimization, Computer Journal, pp. 308-313, 1965.

- 
- [3]Z. Chen, L. Wu, P. Lin, Y. Wu and S. Cheng, Parameters identification of photovoltaic models using hybrid adaptive Nelder-Mead simplex algorithm based on eagle strategy, *Applied Energy*, vol. 182, pp. 47-57, 2016.
  - [4]Sudarshan R. Nelatury. A maximum power point algorithm using the Lagrange method, *Journal of Power Sources* 234 (2013) 119e128.



# Homotopy Iterative Splitting Method to solve the generalized Fisher's Equation

Dadang Amir Hamzah<sup>1</sup> and Eugénio Rocha<sup>2</sup>

<sup>1</sup>Analysis and Geometry Research Division, Institut Teknologi Bandung (ITB), Indonesia

<sup>2</sup>The Center for Research Development in Mathematics and Applications (CIDMA),  
Universidade de Aveiro, Portugal

**Corresponding Author:** *dadang.amir12@gmail.com*

## Abstract

Generalized Fisher's equation is the class of nonlinear reaction diffusion equation where the nonlinearity exist at the reaction part. To solve the problem we use the method so called homotopy iterative splitting. This method is a combination of Iterative Splitting Method and Homotopy Analysis Method. This method transform the nonlinear equation into an infinite (triangular) system of linear differential equation, therefore the nonlinear problem can be solved as the linear problem. Two benchmark problem of Fisher's equation are exhibited to compare the resulted solution with the exact solution.

**Keywords:** Homotopy Iterative Splitting Method, Generalized Fisher's Equation, Reaction Diffusion Equation.

## Acknowledgements

Travel fund is supported by The Center for Research Development in Mathematics and Applications (CIDMA), Universidade de Aveiro, Portugal and The Indonesian Ministry of Research and Technology, Resources Division.

## References

- [1]J. Geiser, *Decomposition Methods for Differential Equations: Theory and Application*, CRC Press, Taylor and Francis Group, 2008.
- [2]A. Molahabrami, F. Khani, *The homotopy analysis method to solve the Burgers-Huxley equation*, Nonlinear Analysis: Real World Applications 10 (2009) 599
- [3]Farago, I., *A modified iterated operator splitting method*, Applied Mathematical Modelling 32 (2008) 1542 - 1551.
- [4]S. Liao, *Beyond Perturbation* Chapman & Hall/CRC, 2004.

## Training Young Researchers in “Pharmaceutical Supply Chains-Medicines Shortages”

João Miranda<sup>1,2</sup>, Cristina Dias<sup>1,3</sup> and Maria José Varadinov<sup>1,4</sup>

<sup>1</sup>Escola Superior de Tecnologia e Gestão do Instituto Politécnico de Portalegre, Portugal

<sup>2</sup>CERENA – Centro de Recursos Naturais e Ambiente do Instituto Superior Técnico, Portugal

<sup>3</sup>CMA – Centro de Matemática e Aplicações da Universidade Nova de Lisboa, Portugal

<sup>4</sup>C3i – Coordenação Interdisciplinar para a Investigação e a Inovação, Portugal

**Corresponding Author:** *jlmiranda@estgp.pt*

### Abstract

A pair of Training Schools (TS) addressing Pharmaceutical Supply Chains (PharmSC) was organized by Instituto Politécnico de Portalegre (IPP), Centro de Recursos Naturais e Ambiente of the Instituto Superior Técnico (CERENA/IST), and IBM Portugal, on behalf of the COST Action European Medicines Shortages Research Network - addressing supply problems to patients, “Medicines Shortages” (CA15105). The first edition took place at IST, Lisboa, Portugal, 26-28 of April-2017, and introductory topics were addressed [1,2], namely, the Action “Medicines Shortages” and the key points (goals, methodology, work-plan, and tools). The second edition was held at Escola Superior de Tecnologia e Gestão (ESTG/IPP), Portalegre, Portugal, 03-07 of July-2017, with the purpose to discuss more advanced topics on Pharmaceutical SC, both on disruptions and on shortages. Young researchers were also invited to present their works, orally on a special session and by poster, being these communications collected and prepared for publication. In addition, the TS main topics are well appreciated, so as the technical sessions and case studies [3], e.g., on Suppliers Selection. Specific computational issues were treated, including SC optimization and IBM Watson/Bluemix for data science experiments, this way spurring joint developments with young researchers and advanced tools.

**Keywords:** : COST Action; Medicines Shortages; Pharmaceutical Supply Chains; Case Studies; Young Researchers.

### Acknowledgements

This communication is based upon work from COST Action CA 15105, European Medicines Shortages Research Network - addressing supply problems to patients (Medicines Shortages), supported by COST (European Cooperation in Science and Technology). Authors also thank: Escola Superior de

Tecnologia e Gestão and Instituto Politécnico de Portalegre; CERENA/IST and the support of FCT-Fundação para a Ciência e a Tecnologia through the project UID-ECI-04028-2013; and Centro de Matemática e Aplicações, Universidade Nova de Lisboa (CMA) through the FCT project UID-MAT-00297-2013.

## References

- [1]Póvoa, A., Corominas, A., and Miranda, J.L. (2016) Optimization and DSS for SC, Springer.
- [2]Miranda, J.L. (2013) Odss.4SC: Optimization and DSS for Supply Chains. In: Mobility, Projects and Cooperation: The LLP best practices (2007-2012). ANPALV, Lisboa: 22-23.
- [3]Miranda, J.L., Varadinov, M.J., and Rubio, S. (2014) Green Logistics, Reverse Logistics and Agro-Industries: Overview of scientific articles and international programs. Ariel, C. and Castro, W. (Ed.) Green Supply Chains: Applications in Agroindustries, Universidad Nacional de Colombia, Manizales: 97-106

## Spectral Clustering Tools for Analysis of Learning Trajectories in the Student Network of Évora University

Juan Luis García Zapata<sup>1</sup>, Maria Clara Grácio<sup>1</sup> and Irene Rodrigues<sup>3</sup>

<sup>1</sup>Universidade de Évora, Departamento de Matemáticas, Espanha

<sup>2</sup>Universidade de Évora, Departamento de Matemática and CIMA, Portugal

<sup>3</sup>Universidade de Évora, Departamento de Informática and LISP, Portugal

**Corresponding Author:** *jjzapata@unex.es*

### Abstract

The study of complex systems through weighted graphs has proved to be very useful, especially in the analysis of social networks. Using clustering techniques, communities are detected in networks of friendship or shared interests. It is done also in networks of scientific collaboration or networks of employment and professional services, see [1]. In this work we study a network of this second type, formed by the students and the disciplines that they have cursing in the e-learning system of the University of Évora. We apply a spectral clustering tool that we have developed, based on the second eigenvector of the Laplacian matrix of the graph. This technique allows to avoid the high cost of combinatorial algorithms using numerical methods of linear algebra, well established in scientific computation, see [2]. In the case under study, the detection of communities identifies trends (such as training profiles that are frequently chosen) and to compare these data with the usual metrics in learning analytics such as performance, study leaving, or repetition rates. In addition to this trajectory detection, our technique can help to the university manager to decide on the investment of resources (mainly attention, guidance and tutoring) over students according to their community profile needs.

**Keywords:** Spectral Clustering, Fiedler Eigenvector, e-Learning, Social Networks.

### Acknowledgements

This work has been partially supported by Centro de Investigação em Matemática e Aplicações (CIMA) through the grant UID/MAT/04674/2013, by Laboratório de Informática, Sistemas e Paralelismo (LISP) through the grant UID/CEC/4668/2016, both research centers are supported by FCT (Fundação para a Ciência e a Tecnologia, Portugal) and, also, by Departamento de Matemáticas, y Escuela Politécnica de Cáceres, de la Universidad de Extremadura, Spain.

## References

- [1]Newman, M., Barabasi, A.L., and Watts, D.J. *The structure and dynamics of networks*. Princeton University Press, 2011.
- [2]Kannan, R., Vempala, S., Vetta, A. (2004). On Clusterings: Good, Bad and Spectral. *Journal of the ACM*, v.51, pp. 497–515.

## Regional enlarged observability for parabolic semi-linear systems

Hayat Zouiten<sup>1</sup> and Ali Boutoulout<sup>1</sup>

<sup>1</sup>TSI Team, MACS Laboratory, Moulay Ismail University, Faculty of Sciences, Meknes, Morocco

**Corresponding Author:** *hayat.zouiten@yahoo.fr*

### Abstract

The aim of this paper is to study the problem of the enlarged observability for distributed parabolic semilinear systems evolving in spatial domain  $\Omega$ . We will explore an approach based on the Hilbert Uniqueness Method (HUM), that can reconstruct the initial state between two prescribed functions  $f_1$  and  $f_2$  only in a critical subregion  $\omega$  of  $\Omega$  without the knowledge of the state. Finally, the obtained results are illustrated by numerical simulations.

**Keywords:** Distributed systems, parabolic systems, semi-linear systems, regional observability, HUM approach.

### Acknowledgements

This work has been carried out with a grant from Hassan II Academy of Sciences and Technology.

### References

- [1]A. EL JAI, M.C SIMON AND E. ZERRIK (1993), *Regional observability and sensors structures*, Sensors and Actuators Journal, Vol 39, 95-102.
- [2]J.L. LIONS AND E. MAGENES (1968), *Problèmes aux limites non homogènes et applications*, Vol 1 et 2, Dunod, paris.
- [3]J.L. LIONS (1988), *Contrôlabilité exacte perturbations et stabilisation des systèmes distribués*, Tome 1, contrôlabilité exacte, Masson, Paris.
- [4]J.L. LIONS (1989), *Sur la contrôlabilité exacte élargie*, Progress in Nonlinear Differential Equations and Their Applications, Vol 1, 703-727.
- [5]E. ZERRIK, H. BOURRAY AND A. BOUTOULOUT (2002), *Regional Boundary Observability: A Numerical Approach*, Int. J. Appl. Math. Compat. Sci, Vol 12, No 35, 143-151.
- [6]M. BAROUN AND B. JACOB (2009), *dmissibility and observability of observation operators for semilinear problems*, A, Integral Equations Operator Theory 64, 1-20.

## Comparison of means through GLM - An example in Oral Health

J.L. Pereira<sup>1</sup>, L. Mendes<sup>1</sup> and T.A. Oliveira<sup>2,3</sup>

<sup>1</sup>Department of Periodontology, School of Dental Medicine of Porto University, Portugal

<sup>2</sup>Open University Portugal

<sup>3</sup>Centro de Estatística e Aplicações (CEAUL), Lisbon, Portugal

**Corresponding Author:** *jpereiramail@fmd.up.pt*

### Abstract

The periodontitis is the most severe form of periodontal disease and the leading cause of tooth loss in adults and is characterized by the destruction of tooth supporting structures, such as alveolar bone and periodontal ligament [1, 2]. Severe periodontitis is defined by a clinical adherence level larger than 5 mm and/or alveolar bone level located at the apical third of the dental root and moderate periodontitis by a clinical adherence level between 1 and 2 mm and/or bone level located at the intermediate third of the root [3]. Considering this classification of periodontitis it seems evident that patients diagnosed with severe periodontitis lose more teeth than patients with moderate periodontitis over the same period of time. The comparison of the number of teeth between patients with severe periodontitis and those with moderate periodontitis implies that we consider the distributional assumptions made about the variable of interest. In this paper we explore the relevance of the modelling of the number of teeth lost due to periodontitis during in the period from the beginning of treatment to being considered stabilized. The data presentend in this paper were result from a retrospective study with data from the clinical records and orthopantomographies of patients from School of Dental Medicine of Porto, after the required authorization of the ethic committee. The statistical methods adopted in this work were those described by Lindsey and Jones [5]. The data was processed with the packages: *fitdistrplus*, *gamlss*, *lsr*, *MASS*, *goft*. The comparison of means when the variable distribution is skewed can be a complex process, requiring the modulation of the mean. The use of models to compare means includes find the best theoretical distribution that fit the empirical data [7], yet precautions must be taken in the interpretation of the models and restrict criteria must be applied to select the best fit.

**Keywords:** Linear oscillators, Cantor set, Spectrum, Iteration.

### Acknowledgements

This work was supported by FCT - Fundação para a Ciência e a Tecnologia under the project PEst-OE-MAT-UI0117-2014.

## References

- [1]R. J. Genco, W. S. Borgnakke, *Periodontology 2000*, Risk factors for periodontal disease., 62 (1), pp. 59–94 (2013).
- [2]P. Axelsson, J. Lindhe, *Journal of Clinical Periodontology*, Effect of controlled oral hygiene procedures on caries and periodontal disease in adults. 5 (2), pp.133-51 (1978).
- [3]G. C. Armitage, *Periodontology 2000*, Periodontal diagnoses and classification of periodontal diseases. 2, pp. 9-21 (2004).
- [4]R Core Team R, A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL <http://www.R-project.org/> (2014).
- [5]J. K. Lindsey and B. Jones, *Statistics in Medicine*, Choosing among generalized linear models applied to medical data. 17, pp. 59-68 (1998).
- [6]J. W. Osborne, *Practical Assessment, Research Evaluation, Improving your data transformation: Applying the Box-Cox transformation*. 15 (2), <http://pareonline.net/pdf/v15n12.pdf> (2010).
- [7]M. L. Delignette-Muller and C. Dutang, *Journal of Statistical Software*, *fitdistrplus: An R Package for Fitting Distributions*, 64 (4), <https://www.jstatsoft.org/article/view/v064i04/v64i04.pdf> (2025).



## A Genetic Algorithm tweak for result improvement in inverse optimization problems

Sérgio Cavaleiro Costa<sup>1</sup>, Fernando M. Janeiro<sup>1,2</sup> and Isabel Malico<sup>1,3</sup>

<sup>1</sup>Universidade de Évora, Portugal

<sup>2</sup>Instituto de Telecomunicações, Portugal

<sup>3</sup>LAETA, IDMEC, Instituto Superior Técnico, Universidade de Lisboa, Portugal

**Corresponding Author:** *sergiocavaleirocosta@gmail.com, fmtj@uevora.pt, imbm@uevora.pt*

### Abstract

The decrease of global greenhouse gas (GHG) emissions is one of the ways to contain global warming. Through Anaerobic Digestion (AD) organic effluents are transformed into biomass and, in the process, biogas (methane and carbon dioxide) is released. Methane, with a higher GHG potential than CO<sub>2</sub>, is an important contributor to climate change. Therefore, the controlled use of microbes to synthesize organic material and minimize the methane release to the atmosphere with the subsequent methane capture and reutilization is one attractive choice in industries with large organic waste production [1]. Different models were developed to simulate AD. The most common are nonlinear dynamic systems composed of a set of ordinary differential equations. They differ in the number of processes considered. A review of the models used can be found in [2], [3] and [4]. For the purpose of this work, the two step model from Campestrini et al. [5] is used. In order to have a valid model, so it can be used for control purposes, for example, the dynamical model parameters require an estimation. For that reason, an Inverse Optimization (IO) must be performed. Due to the simplicity, flexibility and global search efficiency of Genetic Algorithms (GA) they are largely used in different research areas. However, the conventional implementation of Genetic Algorithms, called here Basic Genetic Algorithms (BGA), faces some difficulties in solving this kind of IO problem. To deal with these problems, a tweak to the BGA is proposed, the Neighbored Genetic Algorithm (NGA). In the newly proposed NGA method, one or more subjects within the population are selected for use in an inner loop of the algorithm. In this loop, those subjects will randomly generate a subpopulation, with a specific number of individuals, from a normal distribution (or other), whose mean is the value of the selected subject. The best subject of this subpopulation will replace the one that generated him, if he is fitter. This will, in principle, enhance the chances of getting new subpopulations closer to the solution. To validate and test the

proposed model, a benchmark function (Goldstein-Price) was used. In this case, the NGA method converged in 99% of the runs, while the BGA method only converged in 38% of the cases. Finally, simulated data for methane production were used in the calibration of the model of Campestrini et al. [5]. In this IO problem, both BGA and NGA were run 100 times in order to compare their performance. After 10 000 iterations, the cost function values for the BGA and NGA models were  $8 \times 10^{-3}$  and  $1 \times 10^{-4}$ , respectively. Even though the new approach has proved to be computationally more expensive per iteration, a lower cost function value with less computational time was consistently found in the 100 tests performed when the NGA method was used.

**Keywords:** Genetic Algorithms, Anaerobic Digestion, Inverse Optimization, Biogas.

### Acknowledgements

Isabel Malico acknowledges the funding provided by FCT, Fundação para a Ciência e Tecnologia (Project UID-EMS-50022-2013).

### References

- [1]Ferreira, M., Marques, I.P. and Malico, I. (2012). Biogas in Portugal: status and public policies in a European context. *Energy Policy*, 43, 267-274.
- [2]Appels, L., Baeyens, J., Degreè, J. and Dewil, R. (2008). Principles and potential of the anaerobic digestion of waste-activated sludge. *Progress in Energy and Combustion Science*, 34(6), 755-781.
- [3]Donoso-Bravo, A., Mailier, J., Martin, C., Rodríguez, J., Aceves-Lara, C. A. and Wouwer, A.V. (2011). Model selection, identification and validation in anaerobic digestion: a review. *Water Research*, 45(17), 5347-5364.
- [4]Bernard, O., HadjSadok, Z., Dochain, D., Genovesi, A. and Steyer, J.P. (2001). Dynamical model development and parameter identification for an anaerobic wastewater treatment process. *Biotechnology and Bioengineering*, 75(4), 424-438.
- [5]Campestrini, L., Eckhard, D., Rui, R. and Bazanella, A.S. (2014). Identifiability analysis and prediction error identification of anaerobic batch bioreactors. *Journal of Control, Automation and Electrical Systems*, 25(4), 438-447.

# A New Type of Stability: semi-Hyers-Ulam-Rassias Stability

L.P. Castro<sup>1</sup> and A. M. Simões<sup>1,2</sup>

<sup>1</sup>CIDMA—Centro de Investigação e Desenvolvimento em Matemática e Aplicações,  
Departamento de Matemática, Universidade de Aveiro, Portugal

<sup>2</sup>CMA-UBI—Centro de Matemática e Aplicações da Universidade da Beira Interior,  
Departamento de Matemática, Universidade da Beira Interior, Portugal

**Corresponding Author:** *asimoes@ubi.pt*

## Abstract

We study different kinds of stabilities for some classes of integral and integro-differential equations. In particular, we will introduce the notion of *semi-Hyers-Ulam-Rassias stability*, which is a type of stability somehow in-between the Hyers-Ulam and Hyers-Ulam-Rassias stabilities. This is considered in a framework of appropriate metric spaces in which sufficient conditions are obtained in view to guarantee Hyers-Ulam-Rassias, semi-Hyers-Ulam-Rassias and Hyers-Ulam stabilities for such a class of integral and integro-differential equations. We will consider the different situations of having the integrals defined on finite and infinite intervals. Among the used techniques, we have fixed point arguments, generalizations of the Bielecki metric and Bielecki metric. Some examples of the application of the proposed theory are included.

**Keywords:** Hyers-Ulam stability, semi-Hyers-Ulam-Rassias stability, Hyers-Ulam-Rassias stability, Banach fixed point theorem, Bielecki metric, nonlinear integral equation, integro-differential equation.

## Acknowledgements

This work was supported in part by FCT-Portuguese Foundation for Science and Technology through the Center for Research and Development in Mathematics and Applications of University of Aveiro, within project UID-MAT-04106-2013, and through the Center of Mathematics and Applications of University of Beira Interior, within project UID-MAT-00212-2013.

## References

- [1]Castro, L.P., and Guerra, R.C. (2013) Hyers-Ulam-Rassias stability of Volterra integral equations within weighted spaces, *Lib. Math. (N.S.)* v.33, n.2, pp. 21–35.

- [2]Castro, L.P., and Ramos, A. (2010) Hyers-Ulam and Hyers-Ulam-Rassias stability of Volterra integral equations with a delay, *Integral Methods in Science and Engineering* v.1, pp. 85–94.
- [3]Castro, L.P., and Ramos, A. (2009) Hyers-Ulam-Rassias stability for a class of nonlinear Volterra integral equations, *Banach J. Math. Anal.* v.3, n.1, pp. 36–43.
- [4]Castro, L.P., and Ramos, A. (2011) Hyers-Ulam stability for a class of Fredholm integral equations, *Mathematical Problems in Engineering Aerospace and Sciences ICNPAA 2010, Proceedings of the 8th International Conference of Mathematical Problems in Engineering, Aerospace and Science*, pp. 171–176.
- [5]Castro, L.P., and Simões, A.M. (2017) Hyers-Ulam and Hyers-Ulam-Rassias stability of a class of Hammerstein integral equations, *AIP Conference Proceedings* v.1798:020036, pp. 1–10.
- [6]Castro, L.P., and Simões, A.M. (2017) Hyers-Ulam and Hyers-Ulam-Rassias stability of a class of integral equations on finite intervals, *CMMSE'17: Proceedings of the 17th International Conference on Computational and Mathematical Methods in Science and Engineering* v.I-IV, pp. 507–515.
- [7]Castro, L.P., and Simões, A.M. (2017) Different Types of Hyers-Ulam-Rassias Stabilities for a Class of Integro-Differential Equations, *FILOMAT* (to appear).
- [8]Castro, L.P., and Simões, A.M. (2017) Hyers-Ulam and Hyers-Ulam-Rassias stability for a class of integro-differential equations, in *“Mathematical Methods in Engineering: Theoretical Aspects”*, K. Tas, D. Baleanu and J.A. Tenreiro Machado (eds.), Springer, to appear.
- [9]Xia, C. (2017) Hyers-Ulam stability of the iterative equation with a general boundary restriction, *J. Comput. Appl. Math.* v.322, pp. 7–17.
- [10]Hassan, A.M., Karapinar, E., and Alsulami, H.H. (2016) Ulam-Hyers Stability for MKC Mappings via Fixed Point Theory, *J. Funct. Spaces*, pp. 11.
- [11]Hyers, D.H. (1941) On the stability of linear functional equation, *Proc. Natl. Acad. Sci.* v.27, n.4, pp. 222–224.
- [12]Brzdek, J., Popa, D., and Rasa, I. (2017) Hyers-Ulam stability with respect to gauges, *J. Math. Anal. Appl.* v.453, n.1, pp. 620–628.
- [13]Jung, S.-M. (2007) A fixed point approach to the stability of a Volterra integral equation, *Fixed Point Theory Appl.* v.2007, pp. 9.
- [14]Mohiuddine, S. A., Rassias, J.M., and Alotaibi, A. (2017) Solution of the Ulam stability problem for Euler-Lagrange-Jensen  $k$ -quintic mappings, *Math. Method. Appl. Sci.* v.40, n.8, pp. 3017–3025.
- [15]Rassias, Th. M. (1978) On the stability of the linear mapping in Banach spaces, *Proc. Amer. Math. Soc.* v.72, pp. 297–300.



**Contributed Poster**



## The Image of the Alentejo as a Tourist Destination through the eyes of Lisbon's inhabitants

Luís Sancho<sup>1</sup>, Victor Figueira<sup>1</sup> and M. Teresa Godinho<sup>1</sup>

<sup>1</sup>Escola Superior de Tecnologia e Gestão, Instituto Politécnico de Beja, Portugal

**Corresponding Author:** *luis.sancho@sapo.pt, victorfigueira@ipbeja.pt, mt-godinho@ipbeja.pt*

### Abstract

Lisbon is a market of huge importance for tourism in the Alentejo [1]. On the other hand, it is well known that the image of a tourist destination has a direct influence on its demand [2]. Thus, understanding the image of the Alentejo as a Tourist Destination among Lisbon residents is of major importance to stakeholders. This paper presents a study on the image of the Alentejo as a Tourist Destination, based on the views and opinions of Lisbon's inhabitants. To study these topics, an inquiry by questionnaire was applied to a sample of 400 individuals, stratified by gender, age and education, according to the latest Census available [3]. The sample was a non-probability sample, as, in each category, the members of the sample were selected by convenience. Interviews were conducted in November of 2013 in several environments, all of which with great affluence of the public. The questionnaire was designed to cover both the tangible and the intangible attributes of the destination. Results have shown the role of perceptual/cognitive and affective assessment in the construction of the image of Alentejo. An update on these results is currently taking place. This update will allow to access the impact of the recent communicational strategy used by Turismo do Alentejo.

**Keywords:** Tourist Destination Image, Image Perceptions, Inquiry by Questionnaire.

### Acknowledgements

### References

- [1]TURISMO DO ALENTEJO – ERT .2012. Caracterização do perfil do visitante.
- [2]LIN, C; HUANG, Y. 2009. "Mining tourist imagery to construct destination image position model". Expert Systems with Applications, Vol. 36, pp. 2513-2524.
- [3]INE. 2011.Censos - Resultados definitivos. Região de Lisboa.



## Multivariate APC model in the analyze of the logistics activities of companies that implement or not a system of reverse logistics

Maria José Varadinov<sup>1,2</sup>, Nicolau Almeida<sup>1,2</sup>, João Romacho<sup>1,2</sup>,  
Cristina Dias<sup>3,5</sup> and Carla Santos<sup>4,5</sup>

<sup>1</sup>Instituto Politécnico de Portalegre, Departamento de Ciências Económicas e das Organizações, Portugal

<sup>2</sup>Coordenação Interdisciplinar para a Investigação e a Inovação (C3i), Portugal

<sup>3</sup>Instituto Politécnico de Portalegre, Departamento de Tecnologias, Portugal

<sup>4</sup>Instituto Politécnico de Beja Departamento de Matemática e Ciências Físicas, Portugal

<sup>5</sup>Centro de Matemática e Aplicações da Universidade Nova de Lisboa (CMA), Portugal

**Corresponding Author:** *dinov@estgp.pt, nicolau@estgp.pt, jromacho@estgp.pt, cpsd@estgp.pt, carla.santos@ipbeja.pt*

### Abstract

A reverse logistics system defines a supply chain that is redesigned to efficiently manage the flow of products and parts intended for reprocessing, recycling or disposal. Knowing the importance that global organizations attach to environmental protection and food quality, the wine and olive industries need to follow specific procedures at strategic and operational level. The return of the bottled wine, having reached the expiration date or change the quality, which influences the quality perceived by retail customers, especially the HORECA channel distribution (consisting of hotels, restaurants and cafes), requires adoption of a reverse logistics systems and wine and olive oil producers have an interest in finding a centralized solution that adds value to these products. This study intended to analyze the logistics activities of companies that implement or not a reverse logistics system and understand the economic, social and legislative factors that significantly determine that adoption in the companies with production facilities for wine and olive oil in the Alentejo region. A critical analysis of the variables that affect the reverse logistics as well as their interactions can be quite valuable as an important source of information for decision makers.

**Keywords:** HORECA channel, reverse logistics, wine industry.

### Acknowledgements

Research partially funded by FCT Fundação para a Ciência e a Tecnologia, Portugal, through the projects PEst-OE-MAT-UI0006-2014, UID-MAT-00006-2013.

## References

- [1]Beamon, B.M. (1999). Designing the Green Supply Chain, *Logistics Information Management*, 12(4), 332-342.
- [2]Bollen, K. (1989). *Structural Equations with Latent Variables*. John Wiley and Sons, Inc., New York.
- [3]González-Benito, J. et al.(2006). The role of stakeholder pressure and managerial values in the implementation of environmental logistics practices. *International Journal of Production Research*, 44(7), 1353-1373.
- [4]Manual de Boas Práticas de Eficiência Energética (2005) – Conselho Empresarial para o Desenvolvimento Sustentável (BCSD Portugal).
- [5]Murphy, P. e Poist, R. (2003). Green perspectives and practices: a “comparative logistics” study. *Supply Chain Management: An International Journal*, 8(2), 122-131.
- [6]Wu, H., Dunn, S. (1994), Environmentally responsible logistics systems, *International*

## Symmetric Stochastic Matrices

Cristina Dias<sup>3,5</sup>, Carla Santos<sup>4,5</sup>, João Romacho<sup>3,2</sup>, Maria José Varadinov<sup>3,2</sup> and João Tiago Mexia<sup>1,5</sup>

<sup>1</sup>Departamento de Matemática da Universidade Nova de Lisboa, Portugal

<sup>2</sup>Coordenação Interdisciplinar para a Investigação e a Inovação (C3i), Portugal

<sup>3</sup>Instituto Politécnico de Portalegre, Departamento de Tecnologias, Portugal

<sup>4</sup>Instituto Politécnico de Beja Departamento de Matemática e Ciências Físicas, Portugal

<sup>5</sup>Centro de Matemática e Aplicações da Universidade Nova de Lisboa (CMA), Portugal

**Corresponding Author:** *jtm@fct.unl.pt, dinov@estgp.pt, jromacho@estgp.pt, cpsd@estgp.pt, carla.santos@ipbeja.pt*

### Abstract

The models we developed are  $k$ -degree models of the form  $M = \mu + \bar{E}$  for symmetric stochastic matrices  $M$ , with mean matrix  $\mu$ , and  $E$  is a symmetric stochastic matrix with null mean, and the degree of  $M$  is  $k = \text{car}(\mu)$ . The models are developed using the spectral analysis of the matrices  $\mu$ . The adjustment and validation of the model requires the usage of the vector  $\tilde{\beta}$ , which is an estimator of the structure vector  $\beta$  of  $M$ . For the models with a degree  $k > 1$ , we still consider the possibility of truncating the model, when there are eigenvalues  $\theta_1, \dots, \theta_k$  much greater than the other myth.

**Keywords:** Models, Symmetric Stochastic Matrices, Degree, Eigenvalues.

### Acknowledgements

Research partially funded by FCT Fundação para a Ciência e a Tecnologia, Portugal, through the projects PEst-OE-MAT-UI0006-2014, UID-MAT-00006-2013.

### References

- [1]Areia A. (2009). Séries emparelhadas de estudos. Ph.D. Thesis. Évora University.
- [2]Areia A. (2009). Séries emparelhadas de estudos. Ph.D. Thesis. Évora University.
- [3]J. T. Mexia (2009). Standardized Orthogonal Matrices and Decomposition of the Sum of Squares for Treatments. Trabalhos de Investigação, n°2, Departamento de Matemática, Faculdade de Ciências e Tecnologia da Universidade Nova de Lisboa.
- [4]C. Dias (2013). Modelos e Famílias de Modelos para Matrizes Estocásticas Simétricas. Ph.D. Thesis, Évora University.
- [5]M. M. Oliveira and J. T. Mexia (1999). F Tests for Hypothesis on the Structure Vectors of Series. *Discussiones Mathematicae*, 19(2), 345-353.
- [6]H. Scheffé (1959). *The Analysis of Variance*, New York: John Wiley & Sons.

## PDLs and Flowcharts in Operator Theory

Ana C. Conceição<sup>1,2</sup>

<sup>1</sup>Centro de Análise Funcional, Estruturas Lineares e Aplicações (CEAFEL), Portugal

<sup>2</sup>Departamento de Matemática, Faculdade de Ciências e Tecnologia, Universidade do Algarve, Portugal

**Corresponding Author:** *aicdoisg@gmail.com*

### Abstract

In recent years, several software applications were made available to the general public with extensive capabilities of symbolic computation. These applications, known as computer algebra systems (CAS), allow to delegate to a computer all, or a significant part, of the symbolic calculations present in many mathematical algorithms. The main goal of this work is to present some Program Design Languages (PDLs) and flowcharts developed by us, and others, within Operator Theory.

**Keywords:** Operator theory, symbolic computation, Program Design Languages, flowcharts.

### Acknowledgements

This research is supported by Fundação para a Ciência e Tecnologia (Portugal) through the Center for Functional Analysis, Linear Structures and Applications.

### References

- [1]Conceição, A.C., Kravchenko, V.G., and Pereira, J.C. (2013) Computing some classes of Cauchy type singular integrals with Mathematica software, *Adv.Comput.Math.*, v.39(2), pp. 273–288.
- [2]Conceição, A.C., Marreiros, R.C., and Pereira, J.C. (2016) Symbolic computation applied to the study of the kernel of a singular integral operator with non-Carleman shift and conjugation, *Math.Comput.Sci.*, v.10(3), pp. 365–386.
- [3]Conceição, A.C., and Pereira, J.C. (2016) Exploring the spectra of some classes of singular integral operators with symbolic computation, *Math.Comput.Sci.*, v.10(2), pp. 291–309.

## Building up complex models with commutative orthogonal block structure

Carla Santos<sup>4,5</sup>, Célia Nunes<sup>3</sup>, Cristina Dias<sup>3,5</sup> and João Tiago Mexia<sup>1,5</sup>

<sup>1</sup>Departamento de Matemática da Universidade Nova de Lisboa, Portugal

<sup>2</sup>Coordenação Interdisciplinar para a Investigação e a Inovação (C3i), Portugal

<sup>3</sup>Department of Mathematics and Center of Mathematics–University of Beira Interior, Portugal

<sup>4</sup>Instituto Politécnico de Beja Departamento de Matemática e Ciências Físicas, Portugal

<sup>5</sup>Centro de Matemática e Aplicações da Universidade Nova de Lisboa (CMA), Portugal

**Corresponding Author:** *jtm@fct.unl.pt, cpsd@estgp.pt, carla.santos@ipbeja.pt, celian@ubi.pt*

### Abstract

Linear mixed models provide a flexible approach in situations of correlated data, for example, due to repeated measurements in experiments, see [7], in biology, medical research, animal and human genetics, agriculture or industry. In this work we are interested in a special class of mixed models, those with Commutative Orthogonal Block Structure, COBS, [11], [6]. Models with orthogonal blocks structure, OBS, introduced in [9] and [10] are characterized by their variance–covariance matrices being all positive semi-definite linear combinations of known pairwise orthogonal orthogonal projection matrices, POOPM, whose sum is the identity matrix. When the variance–covariance matrix commutes with the orthogonal projection matrix on the space spanned by the mean vector, the OBS is called COBS (model with commutative orthogonal block structure). This special class of OBS, was introduced in [8] and have been considered too by [12], [2], [3], [4] and [1]. The commutativity condition of COBS is a necessary and sufficient condition for the least square estimators to be best linear unbiased estimators, whatever the variance components, [14]. To build up complex models from simpler ones we may consider the operations, introduced in see [8], named models crossing and models nesting, based on the Kronecker product of commutative Jordan algebras of symmetric matrices, CJAS, and the restricted Kronecker product of CJAS [5]. Crossing models consists of taking two models and obtaining a new model whose treatments are all the possible combinations of those on the two initial models. In model nesting, each treatment of a model nests all the treatments of another model. An alternative to these operations is models joining [13]. Joining  $n$  initial models, we obtain a new model in which the observations vector is the overlap of

observations vectors of the initial models. The technic used to join models rests on the algebraic structure of COBS and the Cartesian product of CJAS [4]. We prove that joining COBS we obtain a new COBS.

**Keywords:** Commutative Jordan algebra, commutative orthogonal block structure, inference, joining models, linear mixed models.

### Acknowledgements

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the projects UID-MAT-00297-2013 and UID-MAT-00212-2014.

### References

- [1]Bailey , R.A., Ferreira, S.S., Ferreira, D., Nunes, C. (2015) Estimability of variance components when all model matrices commute, *Linear Algebra and its Applications*, v.492, pp.144–160.
- [2]Carvalho, F., Mexia, J.T., Oliveira, M.M. (2008) Canonic inference and commutative orthogonal block structure, *Discussiones Mathematicae – Prob. and Stat.*, v. 28, n.2, pp. 171–181.
- [3]Carvalho, F., Mexia, J. T., Santos, C. and Nunes, C. (2015). Inference for types and structured families of commutative orthogonal block structures, *Metrika*, v.78, pp.337–372.
- [4]Ferreira, S.S. , Ferreira, D., Nunes, C. and Mexia, J.T. (2013) Estimation of Variance Components in Linear Mixed Models with Commutative Orthogonal Block Structure, *Revista Colombiana de Estadística* v.36, n.2, pp.259–269.
- [5]Fonseca, M., Mexia, J.T. and Zmyslony, R. (2006). Binary Operations on Jordan algebras and orthogonal normal models, *Linear Algebra and its Applications* v.117, n.1, pp.75–86.
- [6]Fonseca, M., Mexia, J.T. and Zmyslony, R. (2008). Inference in normal models with commutative orthogonal block structure, *Acta et Commentationes Universitatis Tartuensis de Mathematica*, v.12, pp.3–16.
- [7]Khuri, A. I., Mathew, T. and Sinha, B.K. (1998) *Statistical Tests for Mixed Linear Models*, New York, Wiley.
- [8]Mexia, J.T.; Vaquinhas, R.; Fonseca, M. and Zmyslony, R. (2010) COBS: segregation, matching, crossing and nesting. *Latest Trends and Applied Mathematics, Simulation, Modelling, 4-th International Conference on Applied Mathematics, Simulation, Modelling (ASM'10)*, pp. 249–255.
- [9]Nelder J A (1965) The analysis of randomized experiments with orthogonal block structure I. Block structure and the null analysis of variance, *Proceedings of the Royal Society, Series A*, v.283, pp.147–162.
- [10]Nelder J A (1965) The analysis of randomized experiments with orthogonal block structure II. Treatment structure and the general analysis of variance, *Proceedings of the Royal Society, Series A*, v.283, pp.163–178.
- [11]Nunes C., Santos C., Mexia, J. T. (2008) Relevant statistics for models with commutative orthogonal block structure and unbiased estimator for variance components, *Journal of Interdisciplinary Mathematics* v. 11, n.4, pp. 553–564.
- [12]Santos C., Nunes C. and Mexia J. T. (2007) OBS, COBS and Mixed Models associated to commutative Jordan Algebra, *In proceedings of 56th session of the International Statistical Institute*, Lisbon.

- [13] Santos C., Nunes C., Dias, C. and Mexia J. T. (2017) Joining models with commutative orthogonal block structure. *Linear Algebra and its Applications*, v.517. pp.235–245.
- [14] Zmysłony, R., (1978) A characterization of best linear unbiased estimators in the general linear model, *Mathematical Statistics and Probability Theory*, v.2, pp.365–373.

## Condensing normal OBS

Carla Santos<sup>4,5</sup>, Célia Nunes<sup>3</sup>, Cristina Dias<sup>3,5</sup> and João Tiago Mexia<sup>1,5</sup>

<sup>1</sup>Departamento de Matemática da Universidade Nova de Lisboa, Portugal

<sup>2</sup>Coordenação Interdisciplinar para a Investigação e a Inovação (C3i), Portugal

<sup>3</sup>Department of Mathematics and Center of Mathematics–University of Beira Interior, Portugal

<sup>4</sup>Instituto Politécnico de Beja Departamento de Matemática e Ciências Físicas, Portugal

<sup>5</sup>Centro de Matemática e Aplicações da Universidade Nova de Lisboa (CMA), Portugal

**Corresponding Author:** *jtm@fct.unl.pt*, *cpsd@estgp.pt*, *carla.santos@ipbeja.pt*, *celian@ubi.pt*

### Abstract

Correlated data arise frequently in statistical analyses. This may be due, for example, to grouping of subjects or to repeated measurements on each subject over time or space. In these situations, mixed models provide a general, flexible approach. In this work we will focus on models with orthogonal block structure (OBS), a special class within the mixed linear models. A linear mixed model has orthogonal block structure when its variance-covariance matrices are all the linear combinations of known pairwise orthogonal orthogonal projection matrices that add up to the identity matrix. These models were introduced by [5] , [6] and continue to play an important role in the theory of randomized block designs, see [1] , [2]. Commutative Jordan algebras, this is, linear subspaces constituted by symmetric matrices that commute and containing the squares of its matrices, are useful in discussing the algebraic structures of the models in a way that is convenient for deriving estimators both of variance components and estimable vectors through the introduction of sub-vectors [4]. Assuming normality, OBS have complete sufficient statistics and uniformly minimum variance unbiased estimators both for estimable vectors and variance-covariance matrices, [3]. Availing ourselves of the algebraic structure of a normal mixed linear model with orthogonal block structure, resting on commutative Jordan algebras, we can condense this model obtaining a new normal mixed linear model with orthogonal block structure, with less observations than the initial one but ensuring that the good properties on estimation of estimable vectors and variance components continue to hold for the condensed model.

**Keywords:** Commutative Jordan algebra, model condensation, orthogonal block structure, estimation, linear mixed models.



## Acknowledgements

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the projects UID-MAT-00297-2013 and UID-MAT-00212-2014.

## References

- [1]Calinski, T. & Kageyama S. (2000) Block Designs: A Randomization Approach, Vol. I: Analysis, *Lecture Note in Statistics* , n.150, New York: Springer-Verlag.
- [2]Calinski, T. & Kageyama S., (2000) Block Designs: A Randomization Approach, Vol. II: Design, *Lecture Note in Statistics* , n.170, New York: Springer-Verlag.
- [3]Carvalho, F., Zmyslony, R. and Mexia, J.T.(2016) Normal models with Orthogonal Block Structure, *International Journal of Mathematical and Computational Methods* v.1, pp.159–164.
- [4]Ferreira, S., Nunes, C., Ferreira, D., Moreira, E., Mexia, J.T. (2015) Estimation and Orthogonal Block Structure . *Hacettepe University Bulletin of natural Science and Engeneering serie B: Mathematics ans Statistics*, v.45, n.2, pp. 541–548.
- [5]Nelder, J.A. (1965a) The analysis of randomized experiments with orthogonal block structure I. Block structure and the null analysis of variance. Proceedings of the Royal Society, Series A, v.283, pp.147–162.
- [6]Nelder, J.A. (1965b) The analysis of randomized experiments with orthogonal block structure II. Treatment structure and the general analysis of variance. Proceedings of the Royal Society, Series A, v.283, pp.163–178.

## Regression model of sugarcane juice sugar content as a function of the measurement height on the stalk

Dina Mateus<sup>1</sup> and Henrique Pinto<sup>1</sup>

<sup>1</sup>Instituto Politécnico de Tomar, Portugal

**Corresponding Author:** *dinamateus@ipt.pt, hpinho@ipt.pt*

### Abstract

Sugarcane is a valuable crop in the tropical and sub-tropical regions for sugar and ethanol production [1,2]. Sugarcane can also be cultivated in non-tropical countries like Portugal, when integrated in wastewater biological treatment systems [3,4]. Culture control and optimization requires monitoring of several growth indicators, the most important of which is the sugar content [5, 6]. Sugar content of sugarcane plants may be assessed in the field using simple methods based on light refraction by sugarcane juice [7]. Light refraction expressed in Brix degrees (Brix) correlates with the sugar content [7]. However, the Brix readings vary with the height at which measurements are obtained on the sugarcane stalk and previous works concluded that average sugar content may be assessed at the middle internode for sugarcane grown on typical agriculture conditions in tropical land fields [7]. This work studied the dependence of Brix on measurement height in sugarcane (*Saccharum officinarum*) cultivated in biological wastewater treatment systems located in Portugal [8]. Fourteen sugarcane plants were divided in fragments corresponding to the internodes and the Brix was measured for each fragment. Average Brix for each plant was found to approach the Brix near the middle internode (Average relative internode =  $0.565 \pm 0.006$ , 14 data points, passed the Shapiro-Wilk normality test,  $p=0.131$ ), but is statistically different from the value of 0.5 reported for typical production in Brazilian farms (one sample t-test for mean =0.5,  $p < 0.001$ ). A new and more complete model is proposed from the regression of Brix data against relative measurement height (Relative Brix =  $1.284 \pm 0.036 - \text{Relative height} \times 0.497 \pm 0.057$ , 110 data points,  $F < 0.001$ ,  $r^2 = 0.735$ ), which can be used to make fast estimations of sugarcane sugar content from plant samples obtained at any plant height.

**Keywords:** Linear regression, *Saccharum officinarum*, Brix.

**Acknowledgements**

## References

- [1]S. Solomon, M. Swapna, V.T. Xuan and Y.Y. Mon (2016). Development of sugar industry in ASEAN Countries. *Sugar Tech* 18, 559-575.
- [2]A.P. de Souza, A. Grandis, D.C.C. Leite and M.S. Buckeridge (2014). Sugarcane as a bioenergy source: history, performance, and perspectives for second-generation bioethanol. *Bioenergy Research* 7, 24-35.
- [3]D.M.R. Mateus, M. M. Vaz, I. Capela and H. J.O. Pinho (2014). Sugarcane as constructed wetland vegetation: preliminary studies. *Ecological Engineering* 62, 175-178.
- [4]D.M.R. Mateus, M. M. Vaz and H. J.O. Pinho (2017). Valorisation of phosphorus-saturated constructed wetlands for the production of sugarcane. *Journal of Technology Innovations in Renewable Energy* 6, 1-6.
- [5]J.C.S. Allison, N.W. Pammenter and R.J. Haslam (2007). Why does sugarcane (*Saccharum* sp. hybrid) grow slowly? *South African Journal of Botany* 73, 546-551.
- [6]L.C. Tasso Júnior, M.O. Marques, A. Franco, G.A. Nogueira, F.O. Nobile, F. Camilotti and A.R. Silva, (2007). Yield and quality of sugar cane cultivated in sewage sludge, vinasse and mineral fertilization supplied soil. *Engenharia Agrícola* 27, 276-283.
- [7]A. Azzini, J.P.F. Teixeira, R.M. Moraes and J.F.P. Camargo. (1980). Correlation between the soluble solid contents of the cane juice and the culm basic density. *Bragantia* 39, 181-183.
- [8]D.M.R. Mateus, M. M. Vaz, I. Capela and H. J.O. Pinho (2016). The potential growth of sugarcane in constructed wetlands designed for tertiary treatment of wastewater. *Water* 8 (93), 1-14.

## One-dimensional Third-grade Fluid Model

Fernando Carapau<sup>1</sup>, Paulo Correia<sup>1</sup> and Luís M. Grilo<sup>2</sup>

<sup>1</sup>Universidade de Évora, Departamento de Matemática e CIMA, Portugal

<sup>2</sup>Instituto Politécnico de Tomar e CMA/FCT/UNL, Portugal

**Corresponding Author:** *flc@uevora.pt, pcorreia@uevora.pt, lgrilo@ipt.pt*

### Abstract

Three-dimensional numerical simulations of non-Newtonian fluid flows are a challenging problem due to the particularities of the involved differential equations leading to a high computational effort in obtaining numerical solutions, which in many relevant situations becomes infeasible. Several models have been developed along the years to simulate the behavior of non-Newtonian fluids together with many different numerical methods. In this work we use a one-dimensional hierarchical approach to a proposed generalized third-grade fluid with shear-dependent viscoelastic effects model. This approach is based on the Cosserat theory related to fluid dynamics and we consider the particular case of flow through a straight and rigid tube with constant circular cross-section. With this approach, we manage to obtain results for the wall shear stress and mean pressure gradient of a real three-dimensional flow by reducing the exact three-dimensional system to an ordinary differential equation. This one-dimensional system is obtained by integrating the linear momentum equation over the constant cross-section of the tube, taking a velocity field approximation provided by the Cosserat theory. From this reduced system, we obtain the unsteady equations for the wall shear stress and mean pressure gradient depending on the volume flow rate, Womersley number, viscoelastic coefficients and the flow index over a finite section of the tube geometry. Attention is focused on some numerical simulations for constant and non-constant mean pressure gradient using a Runge-Kutta method.

**Keywords:** One-dimensional model, generalized third-grade model, shear-thickening fluid, shear-thinning fluid, Cosserat theory.

### Acknowledgements

The researcher belongs to the Centro de Investigação em Matemática e Aplicações, Universidade de Évora, Project UID-MAT-04674-2013, a research centre supported by FCT (Fundação para a Ciência e a Tecnologia, Portugal).

## References

- [1]Carapau, F., Correia, P., Grilo, L.M., and Conceição, R. (2017) Axisymmetric Motion of a Proposed Generalized Non-Newtonian Fluid Model with Shear-dependent Viscoelastic Effects, *IAENG International Journal of Applied Mathematics*, accepted for publication.
- [2]Caulk, D.A., and Naghdi, P.M. (1987) Axisymmetric motion of a viscous fluid inside a slender surface of revolution, *Journal of Applied Mechanics*, v.54, n.1, pp. 190–196.
- [3]Fosdick, R.L., and Rajagopal, K.R. (1980) Thermodynamics and stability of fluids of third grade, *Proc. R. Soc. Lond. A.*, v.339, pp. 351–377.

## Investigating the properties of Rascal's Triangle

Zeferino Caxala<sup>1</sup>, Rogério Seródio<sup>1</sup> and Ilda Inácio Rodrigues<sup>1</sup>

<sup>1</sup>Faculdade de Ciências, Departamento de Matemática, Universidade da Beira Interior,  
Portugal

**Corresponding Author:** *rserodio@ubi.pt, rserodio@ubi.pt, ilda@ubi.pt*

### Abstract

The Rascal's Triangle appeared as a result of “inquiry-based learning that transforms how we think about what we know and how we know. It challenges us to reconsider the nature of teaching and learning in mathematics” [1]. Rascal's Triangle was discovered by A. Anggoro, E. Liu and A. Tulloch, and follows a rule different from that of Pascal's Triangle, although the first four rows are equal. Their rule was that “the outside numbers on each row are 1s and the inside numbers are determined by the diamond formula: South = (West + East + 1) ÷ North” [2]. Little is known about its properties. Our propose is to present what is known up to date and to share some properties that were discovered from our research.

**Keywords:** Rascal's Triangle, Pascal's Triangle, Combinatorics

### Acknowledgements

### References

- [1] Julian Fleron, Tackling Rascals' Triangle - How Inquiry Challenges What We Know and How We Know It, *Discovering the Art of Mathematics*, December 15, 2015.
- [2] A. Anggoro, E. Liu and A. Tulloch, The Rascal Triangle, *College Math. J.*, Vol. 41, No. 5, Nov. 2010, pp. 393-395.

## Prevalence of Pediatric Hypertension: a Preliminary Approach

M. Filomena Teodoro<sup>1,2</sup>, Carla Simão<sup>3</sup>, Margarida Abranches<sup>4</sup>, Sofia Deuchande<sup>5</sup> and Ana Teixeira<sup>6</sup>

<sup>1</sup>CINAV, Center of Naval Research, Naval Academy, Portuguese Navy, Almada, Portugal

<sup>2</sup>CEMAT, Center for Computational and Stochastic Mathematics, Instituto Superior Técnico, Lisbon University, Lisboa, Portugal

<sup>3</sup>Hospital de Santa Maria, Serviço de Pediatria, Lisboa, Portugal

<sup>4</sup>Hospital de D. Estefânia, Serviço de Pediatria, Lisboa, Portugal

<sup>5</sup>Hospital de Cascais, Serviço Pediatria, Cascais, Portugal

<sup>6</sup>Hospital de São João, Serviço Pediatria, Porto, Portugal

**Corresponding Author:** *maria.alves.teodoro@marinha.pt*

### Abstract

Pediatric Hypertension is a serious problem in health. This fact was evidenced by the author of [1]. The fact that pediatric high blood pressure can happen is unknown for the majority of the families. This is also described in [3] where some recommendations of European Society of hypertension are discussed. The evaluation of this problem is determinant, so one can prevention [4]. A study about caregivers literacy was started in [2], where was built a preliminary questionnaire with binary answers was built and applied to the users of regular pediatric consultations from Santa Maria's Hospital. Using several multivariate techniques, the statistical analysis of the results was improved and completed in [6–8]. A more complete questionnaire about pediatric hypertension knowledge was introduced in [5]. This work was extended in [9,10]. A study on prevalence of pediatric hypertension at national level is ongoing. A questionnaire was designed to be answered by caregivers of children and Portuguese teenagers, socio-demographic details are inquired, the medical individual characteristics of children and adolescents are observed by a medical team. The data collection is still ongoing over several regions of Portugal. While this work is not complete, a smaller sample (about 5 hundred of observations) is considered to be analyzed statistically. It is performed a descriptive analysis and applied an analysis of variance. Relationships between socio-demographic variables and blood pressure are evaluated. The statistical approach, estimating a model with relevant information using more elaborate techniques such as generalized linear models is still going on. The study described in this article is a preliminary approach, which will used as basis when the designed sample is complete. Some important issues about pediatric hypertension were obtained, e.g. gender and age are related with

pediatric high blood pressure, reinforcing the importance of a knowledge improvement and prevention measures about pediatric high blood pressure.

**Keywords:** Childhood, caregivers, hypertension, analysis of variance, generalized linear models.

### Acknowledgements

This work was supported by Portuguese funds through the *Center of Naval Research* (CINAV), Portuguese Naval Academy, Portugal and *The Portuguese Foundation for Science and Technology* (FCT), through the *Center for Computational and Stochastic Mathematics* (CEMAT), University of Lisbon, Portugal, project UID-Multi-04621-2013.

### References

- [1] Bassareo, P.P. and Mercurio, G. (2014) Pediatric hypertension: An update on a burning problem. *World Journal of Cardiology* v.6, n.5, pp.253–259.
- [2] Costa, J. (2015) *Hipertensão arterial em idade pediátrica: que conhecimento têm os prestadores de cuidados sobre esta patologia?*, Master Thesis, Medical Faculty, Lisbon University.
- [3] Lurbe, E. and Cifkovic, R.F. (2009) Management of high blood pressure in children and adolescents: recommendations of the European Society of Hypertension. *Journal of Hypertension*, v.27, pp.1719–1742.
- [4] National High Blood Pressure Education Program Working Group on High Blood Pressure in Children And Adolescents (2004) The fourth report on the diagnosis, evaluation and treatment of high blood pressure in children and adolescents, *Pediatrics* v.114, pp.555–576.
- [5] Romana, A. (2017) *Hipertensão Arterial em Pediatria. Um estudo observacional sobre a literacia dos cuidadores*, Master Thesis, Medical Faculty, Lisbon University.
- [6] Teodoro, M. F. and Simão, C. (2017) Perception about Pediatric Hypertension. *Journal of Computational and Applied Mathematics*, v.312, pp.209–215.
- [7] Teodoro, M. F. and Simão, C. (2017) Completing the Analysis of a Questionnaire About Pediatric Blood Pressure. *Transactions on Biology and Biomedicine*, World Sci. Eng. Acad. Soc., v.14, pp.56–64.
- [8] Teodoro, M. F. and Simão, C. (2017) Notes about Pediatrics Hypertension Literacy. *Transactions on Biology and Biomedicine*, World Sci. Eng. Acad. Soc., v.14, pp.89–97.
- [9] Teodoro, M. F., Romana, A. and Simão, C. An Issue of Literacy on Pediatric Hypertension. In T. Simos et al., editors, *Computational Methods in Science and Engineering*. New York, AIP Conference Proceedings. (to appear)
- [10] Teodoro, M. F., Simão, C. and Romana, A. (2017) Questioning Caregivers About Pediatric High Blood Pressure. In Gervasi, O. et al. (Eds.), *Computational Science and Its Applications ICCSA 2017, Lecture Notes in Computer Science*, v.10408(V), pp.1–10.



## Applying statistical methods on the analysis of ecological preferences of the spontaneous flora

Sofia Ramôa<sup>1</sup>, Pedro Oliveira e Silva<sup>1</sup>, Teresa Vasconcelos<sup>2</sup>, Paulo Fortes<sup>2</sup> and João Portugal<sup>1</sup>

<sup>1</sup>IPB, Departamento de Biociências, Escola Superior Agrária de Beja, Portugal

<sup>2</sup>Instituto Superior de Agronomia, Portugal

**Corresponding Author:** *sramoa@ipbeja.pt*

### Abstract

There are several statistical methods which can be used in the study of biodiversity. The method of Ecological Profiles and Mutual Information is one of these methods – non-inferential and non-parametric - presented by [1] and later on developed by [2]. It aims to provide information about species indicator values related to environmental factors [3], under the hypothesis of independence between species and classes of the measured factor. The Ecological Profiles of the species correspond to their distribution by classes of environmental factors, giving useful information about the amplitude of their habitat, specifying their ecological behaviour [3]. This methodology is based on the calculation of the Entropy of species and factors, and of the Mutual Information among species / factors, from which the indicator species are selected for each factor studied. It has the advantage of allowing the establishment of groups of species with the same preferences - ecological groups. Other important advantages are: provide information on sampling quality [4], very important issue given the type of sampling performed in studies of this nature; allow the study of specie by specie and factor by factor. Some authors consider this a precise methodology [5] but suggest that a multifactorial approach is needed in order to overcome the inconvenient that statistically significance is not evaluated. With this purpose we used Canonical Correspondence Analysis [6], an inferential and parametric method, from which we can obtain models for the ecological studied factors. The models were created through the stepwise method. Akaike Information Criterion (AIC), an indicator of the information loss of each of the potential models in relation to the best model, was used to automatic selection of the best model. The model that minimizes the Kulback-Leibler distance (lowest AIC) is the selected model, corresponding to the one with the least loss of information [9], [8]. This heuristic approach that searches for the balance between goodness of fit / complexity of the model, minimizing overfitting, is also based on Information Theory [8]. To establish the significance of the

generated models the ANOVA test is performed. These methodologies were used in the following case study: 100 floristic surveys were carried out on wheat, oats and barley plots, and some ecological variables such as pH, texture and soil phosphorus and potassium content, precipitation and soil type were characterized. An analysis of frequency and abundance of species was performed and the methodology of Ecological Profiles and Mutual Information was applied to data, combined with Canonical Correspondence Analysis method. Data processing was done with Microsoft Office Excel 2007 and the R program [10], using the Biodiversity, Mass and Vegan packages [11]. According to the Method of Ecological Profiles and Mutual Information, the distribution of species is closely related to texture and pH. Application of Akaike information criterion and ANOVA show, from the inferential point of view, the importance of the same discriminants factors in the distribution of spontaneous vegetation.

**Keywords:** Frequency and abundance analysis, Entropy, Information Theory, Canonical Correspondence Analysis, plant ecology.

### Acknowledgements

### References

- [1]Gounot, M. (1958). Contribution a l'étude des groupements végétaux messicoles et rudéraux de la Tunisie. Annual Service Botanique et Agronomic de Tunisie 31, 1-288.
- [2]Godron, M. (1965). Les principaux types de profils écologique. C.N.R.S.-C.E.P.E. Montpellier, 8 p.
- [3]Devineau, J.L. and Fournier, A. (2007). Integrating environmental and sociological approaches to assess the ecology and diversity of herbaceous species in a Sudan-type savanna (Bondoukuy, western Burkina Faso). Flora 202, 350-370.
- [4]Vasconcelos, M.T.C. (1984). Estudos bio-ecológicos das infestantes na cultura do tomateiro. Diss. Mestrado em Produção Vegetal. Instituto Superior de Agronomia, Universidade Técnica. Lisboa, 122 p.
- [5]Fariñas, M.R., Lázaro, N. and Monasterio, M. (2008). Ecología comparada de *Hypericum laricifolium* Juss. y *H. juniperinum* Kunth en el valle Fluvioglacial del Páramo de Mucubají. Mérida, Venezuela. Sociedad Venezolana de Ecología. Ecotrópicos 21(2), 75-88.
- [6]Ter Braak, C.J.F. (1987). The analysis of vegetation-environment relationships by canonic correspondence analysis. Vegetatio 69, 69-77.
- [7]Schaefer, J. (2017). Multivariate applications in ecology. Disponível em: <http://ichthyology.usm.edu/courses/multivariate/schedule.php> (acessado:10/10/2017).
- [8]Provet, D.B., Silva, F.R. and Souza, T.G. (2011). Estatística aplicada à ecologia usando o R. Programa de pós-graduação em Biologia Animal. Universidade Estadual Paulista, S. José do Rio Preto, 122 p.
- [9]Wagenmakers, E. and Farrell, S. (2004). AIC model selection using Akaike weights. Psychonomic Bulletin Review 11(1), 192-196.
- [10]R. Development Core Team (2011). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0, URL <http://www.R-project.org/>.

- [11] Oksanen, J., Blanchet, F.G.; Kindt, R. et.al (2013). *vegan: Community Ecology Package*. R package version 1.17-6. <http://CRAN.R-project.org/package=vegan>

## The relations between work-family conflicts, burnout, and cognitive appraisal: a structural equation modelling

J. Rodrigues<sup>1</sup>, A.M. Gonçalves<sup>2</sup>, S. Faria<sup>2</sup>, A.R. Gomes<sup>3</sup> and C. Simões<sup>4</sup>

<sup>1</sup>Department of Mathematics and Applications. University of Minho, Portugal

<sup>2</sup>Centre of Mathematics, Dep. of Mathematics and Applications, University of Minho, Portugal

<sup>3</sup>School of Psychology, University of Minho, Portugal

<sup>4</sup>School of Nursing, University of Minho, Portugal

**Corresponding Author:** *sfaria@math.uminho.pt*

### Abstract

In this study we used Structural Equation Modelling (SEM) to test the mediating role of cognitive appraisal in the relationship between work-family conflicts and burnout. The total sample consisted of 438 Portuguese teachers who teach from kindergarten through high school and completed an evaluation protocol with measures of work-family conflicts, cognitive appraisal, and burnout. The results confirmed cognitive appraisal partially mediated the relationship between work-family conflicts and burnout. The findings indicated that cognitive appraisal is an important underlying mechanism in explaining adaptation at work.

**Keywords:** Burnout, cognitive appraisal, teachers, work-family conflicts.

### Acknowledgements

The authors acknowledge the financial support of the Portuguese Funds through FCT – Fundação para a Ciência e a Tecnologia, within the Project PEstOE-MAT-UI0013-2017.

### References

- [1]Byrne, B. M. (2010) Structural equation modeling with AMOS: Basic concepts, applications, and programming, New York: Routledge.
- [2]Gomes, A. R., Faria, S. and Gonçalves, A. M. (2013) Cognitive appraisal as a mediator in the relationship between stress and burnout. *Work and Stress*, 27 (4), 351- 367.
- [3]Simões, C., McIntyre, T., and McIntyre, S. (2009). Portuguese adaptation of the Work-Family Conflict Family-Work Conflict scales in Nurses: a Preliminary Study. *Psychology Health*, 24 (1), 364.

## Determining the Number of Components in Mixtures of Linear Mixed Models

Susana Faria<sup>1</sup> and Luísa Novais<sup>2</sup>

<sup>1</sup>Centre of Mathematics, Dep. of Mathematics and Applications, University of Minho, Portugal

<sup>2</sup>Dep. of Mathematics and Applications, University of Minho, Portugal

**Corresponding Author:** *sfaria@math.uminho.pt*

### Abstract

Finite mixture models are a well-known method for modelling data that arise from a heterogeneous population. Within the family of mixture regression models, finite mixtures of linear mixed models have also been applied in different areas of application. They conveniently allow to account for correlations between observations from the same individual and to model unobserved heterogeneity between individuals at the same time. Selecting the correct number of components in mixture model is a problem which has not been satisfactorily resolved. In this study the performance of various model selection methods was investigated in the context of Finite Mixtures of Linear Mixed Models.

**Keywords:** Finite Mixtures of Linear Mixed Models, Model selection, Simulation study.

### Acknowledgements

The authors also acknowledge the financial support of the Portuguese Funds through FCT – Fundação para a Ciência e a Tecnologia, within the Project PEstOE-MAT-UI0013-2017.

### References

- [1]Celeux G., Martin O., Lavergne Ch. (2005) Mixture of linear mixed models for clustering gene expression profiles from repeated microarray experiments. *Statistical Modelling*, 5, 1–25.
- [2]Fruhwirth-Schnatter, S. (2006) *Finite Mixture and Markov Switching Models*. Springer Series in Statistics.
- [3]Young D., Hunter D. R. (2015) Random effects regression mixtures for analyzing infant habituation, *Journal of Applied Statistics*, 42:7, 1421–1441.

## E-Commerce: a statistical approach for supply forecasting

A. Manuela Gonçalves<sup>1</sup> and Andreia Ribeiro<sup>2</sup>

<sup>1</sup>Centro de Matemática, Universidade do Minho, Portugal

<sup>2</sup>Departamento de Matemática e Aplicações, Universidade do Minho, Portugal

**Corresponding Author:** *mneves@math.uminho.pt*

### Abstract

E-Commerce has been increasing rapidly and will obviously be more popular in the future. It is believed that the growth of internet has been impacting on E-Commerce's growth worldwide. This research aims to provide insight into a specific E-Commerce marketplace by performing metrics forecasting. A marketplace directly controls the supply, and it is vitally important that it is able to predict/forecast the behavior of its commercial partners (stores) based on specific metrics, and so it is crucial to determine the initial objectives for each one of those partners. For the purpose of this research, clustering techniques, statistical inference procedures and linear models (by incorporating seasonality and trends) will be used as research methods. The dataset corresponds to the weekly and cumulative values of each metric up to the week under observation in a given season of the year.

**Keywords:** E-Commerce, Cluster analysis, Linear Models, Seasonality, Trend, Forecasting.

### Acknowledgements

The authors acknowledge the financial support of the Portuguese Funds through FCT – Fundação para a Ciência e a Tecnologia, within the Project PEstOE-MAT-UI0013-2017.

### References

- [1]Everitt B.S., Landau S., and Leese M. (2001). Cluster analysis. 4th ed. Arnold, London.
- [2]Faraway J.J. (2009). Linear Models with R. Taylor & Francis, New York.
- [3]Higgins, J.J. (2004). Introduction to Modern Nonparametric Statistics. Duxbury Advanced Series.

## Virtual Instrumentation: Evaluation of a data acquisition

Leonardo Andrade<sup>1,2</sup>, Pedro Gonçalves<sup>1,2</sup>, Mouhadydine Tlemçani<sup>1,2</sup>  
and Fernando Janeiro<sup>1,2</sup>

<sup>1</sup>Universidade de Évora, Portugal

<sup>2</sup>Instituto de Ciências da Terra, Portugal

**Corresponding Author:** *Andrade9653@gmail.com*

### Abstract

Whether in research labs or in industrial areas, Virtual Instrumentation (VI) has assumed an increasing role and is continuously becoming more relevant. This situation can be explained by: the constant fall in the prices of electronic products and components that affect the overall costs, flexibility of these systems and hardware reduction due to the use of powerful signal processing algorithms. In this work using numerical algorithms a comparison of different methods for Analog-to-Digital Converter (ADC) characterization is presented [1,2]. One of the methods is based on Fast Fourier Transform (FFT) [3] while the other one is a statistical approach [4]. The result of those characterizations is an estimation of the ADC's effective number of bits and others important parameters useful for data acquisition.

**Keywords:** ADC, Virtual Instrumentation, Characterization.

### Acknowledgements

The researchers thanks the Instituto de Ciências da Terra laboratory, Évora, Portugal and the program Alentejo 2020, BRO-CQ and IDT-COP-17659 for the support.

### References

- [1] Serra, A. C., Alegria, F., Martins, R., Silva, M. F. (2003). Analog-to-digital converter testing — new proposals, 26, 3–13. [https://doi.org/10.1016/S0920-5489\(03\)00057-6](https://doi.org/10.1016/S0920-5489(03)00057-6)
- [2] Scientist, S. (2015). Enhanced ADC Sine Wave Histogram Test, 6–11.
- [3] Cataldi, G., Negreiros, M., Carro, L., Susin, A. A. (2004). INL and DNL Estimation Based on Noise for ADC Test, 53(5), 1391–1395.
- [4] Martins, R. C. (1999). Automated ADC Characterization Using the Histogram Test Stimulated by Gaussian Noise, 48(2), 471–474.

## Computational vision applied in automotive driving support systems

Ramez Aldwihe<sup>1</sup> and José Saias<sup>1</sup>

<sup>1</sup>Departamento de Informática, Universidade de Évora, Portugal

**Corresponding Author:** *eng.ramez87@hotmail.com, jsaias@uevora.pt*

### Abstract

The objectives of this work are: to build a system which recognizes traffic signs by analyzing the images/video taken with a camera installed in the car. The system includes three stages: Detection, Classification, and Recognition. The data set which I used to train and to test the model is German Traffic Sign Recognition Benchmark (GTSRB) data set [1], a publicly available data set for single-image. The system also used with Portuguese traffic signs database and several examples taken from Portuguese roads are used to demonstrate the effectiveness of the proposed system. Traffic signs are detected by analyzing color information, red and blue, and analyzing the shape of the signs as triangular, squared and circular shapes, contained in the images using OpenCV library. To make the classifier, I used Convolutional Neural Network technique with TensorFlow as a Machine Learning framework. The recognition of traffic signs is done by comparing the data from classification phase with the ones of the database. The results in the classifier are almost 97,7 % [2], and results in detection part are 70% for red and blue traffic signs respectively.

**Keywords:** Traffic sign detection, Image processing, Shape analysis, CNN.

### Acknowledgements

This project is a master thesis in the University of Évora. cooperation with Prof José Saias as advisor of my thesis.

### References

- [1] J. Stallkamp, M. Schlipsing, J. Salmen, and C. Igel, Man vs. computer: Benchmarking machine learning algorithms for traffic sign recognition, *Neural Networks*, vol. 32, pp. 323– 332, 2012.
- [2] K. Simonyan and A. Zisserman, Very deep convolutional networks for large-scale image recognition, *ArXiv preprint arXiv:1409.1556*, 2014.



## Application of data mining techniques to E-learning data

Nuno M. Brites<sup>1,4</sup>, Pedro Melgueira<sup>1,3</sup>, Irene Rodrigues<sup>1,2,3</sup> and Lília Ferreira<sup>1,2,3</sup>

<sup>1</sup>Centro de Tecnologias Educativas, Universidade de Évora, Portugal

<sup>2</sup>Laboratório de Informática, Sistemas e Paralelismo, Universidade de Évora, Portugal

<sup>3</sup>Departamento de Informática, Escola de Ciências e Tecnologia, Universidade de Évora, Portugal

<sup>4</sup>Centro de Investigação em Matemática, Universidade de Évora, Portugal

**Corresponding Author:** *brites@uevora.pt*

### Abstract

The advancement of the technologies related to the internet has enabled E-Learning to gain popularity as a way of transmitting knowledge. Universities and Companies, among others institutions, have been using E-Learning to disseminate educational content to remote locations, reaching out to students, researchers and employees who are physically distant (see, for instance, [1] and [2]). The Moodle platform is an example of a Learning Management Systems (LMS). LMS provide on-line platforms where teachers and trainers can publish content organized into activities, conduct assessments, and other tasks so that the students involved can learn and be assessed. In addition, LMS generates and stores large amounts of data, named Educational Data, from not only user activities but also the LMS itself. In this work we will present some data mining techniques applied to Educational Data. From the Moodle data repository of the University of Évora, we will apply supervised learning techniques with the aim of predicting the students success from their interaction with Moodle. We will also see interesting conclusions when unsupervised learning techniques are applied.

**Keywords:** Big data, classification, data mining, decision trees, learning management systems.

### Acknowledgements

This work has been partially supported by Centro de Investigação em Matemática e Aplicações (CIMA) through the grant UID-MAT-04674-2013 and, also, by Laboratório de Informática, Sistemas e Paralelismo (LISP) through the grant UID-CEC-4668-2016, both research centers are supported by FCT (Fundação para a Ciência e a Tecnologia, Portugal).

## References

- [1]Chen, G. and Liu, C. and Ou, K. and Liu, B. (2000) Discovering decision knowledge from web log portfolio for managing classroom processes by applying decision tree and data cube technology. *Journal of Educational Computing Research*, v.23, pp. 305–332.
- [2]Romero, C. and Ventura, S. (2010) Educational Data Mining: A Review of the State of the Art. *Trans. Sys. Man Cyber Part C*, v.40(6), pp. 601–618.

## Wine quality ratings versus price in The Wine Enthusiast Magazine

António Carloto<sup>1</sup>

<sup>1</sup>Escola Superior Agrária, Instituto Politécnico de Beja, Portugal

**Corresponding Author:** *acarloto@ipbeja.pt*

### Abstract

Do more expensive wines taste better? This question has been addressed frequently (Goldstein et al, 2008; Zelený, 2017) and is of greatest relevance, given the enormous number of wines in the market and the broad range of prices at they are sold. Consumers often rely on wine guides, on paper or on line to make buying decisions, based on - along other factors, like origin and grape variety - the quality ratings given by experts and the reference prices they find there. Some researchers, like Schamel and Anderson (2003), found a positive relationship between the quality ratings reported in these guides and the wines prices. In this study, we used a large data set scraped from the Wine Enthusiast Magazine to access the relationship between prices and quality ratings for wines from different countries, trying to find in what measure the consumer can buy high quality wines spending little money. For this particular study, only the top 10 countries in number of wines reviewed were considered. We found that all the countries have quality ratings with reasonable variability but with similar medians comprised between 85 and 90 (only wines with 80 or more are reviewed in this magazine). Prices have great variability at the top level, but the median falls, for all countries, in the range of 14 to 28\$. When we plotted the aggregated quality ratings for all the 10 countries as the dependent variable of price, we found initially a moderate positive relationship between prices and ratings ( $R = 0.55, p < 2.2 \times 10^{-16}$ ) that, around the 200\$ price, changes to a plateau with a very gentle ascent ( $R = 0.21, p < 1.4 \times 10^{-9}$ ). Breaking those results for each country we saw that for extreme prices, in some countries like US, Italy, Austria and Portugal (but not France) an increase in price tends to have a negative relationship with the ratings. Looking more closely to the Portuguese wines, a local maximum could be detected around 100\$ in the “Excellent” range of quality. This could work as a “sweet spot” price reference for the exigent (and wealthy) consumer. But less affluent consumers do not need to spend so much: you can find 148 Portuguese wines rated as “Excellent” between 7 and 15\$. So, do more expensive wines taste better? For top rated wines (80 – 100), in general yes, but not necessarily so.

**Keywords:** Wine, quality, ratings, buying guides, Wine Enthusiast Magazine, public data sets, Kaggle, R.

### Acknowledgements

I thank Zack Thoutt for putting the data set used in this work publicly available at <https://www.kaggle.com/zynicide/wine-reviews>.

### References

- [1]Goldstein, Robin et al. (2008) Do more expensive wines taste better? Evidence from a large sample of blind tastings, SSE/EFI Working Paper Series in Economics and Finance, No. 700.
- [2]Schamel, G. & Anderson, K. (2003) Wine quality and varietal, regional and winery reputation: Hedonic prices for Australia and New Zealand, *The Economic Record*, 79 (246), 357-369.
- [3]Zelený, J. (2017) A relationship between price and quality rating of wines from the Czech Republic, *Journal of International Food & Agribusiness Marketing*, 29:2, 109-119.

## An overview of the classification of Bicontactual Hypermaps

António Breda D'Azevedo<sup>1</sup> and Ilda Inácio Rodrigues<sup>2</sup>

<sup>1</sup>Universidade de Aveiro, Portugal

<sup>2</sup>Universidade da Beira Interior, Portugal

**Corresponding Author:** *ilda@ubi.pt*

### Abstract

This poster presents an overview of the classification of Bicontactual Hypermaps. In this work we have studied the regular hypermaps: orientable, non-orientable, oriented and pseudo-orientable. Bicontactual hypermaps are hypermaps with the property that each hyperface meets only two others. We will present basic notions in theory of hypermaps, the classification of the bicontactual hypermaps and we reclassify, using our algebraic method, the bicontactual non-orientable hypermaps. In the seventies, Steve Wilson classified the bicontactual maps and, in 2003, Wilson and Breda d'Azevedo classified the bicontactual non-orientable hypermaps. When this property is transferred for hypermaps it gives rise to three types of bicontactuality, according as the two hyperfaces appear around a fixed hyperface. A topological hypermap is a cellular embedding of a connected trivalent graph into a compact and connected surface such that the cells are 3-colored. Or simply, a hypermap can be seen as a bipartite map.

**Keywords:** Hypermap, map, orientable, non-orientable, oriented, regular, bicontactual.

### Acknowledgements

Research partially funded by Center of Mathematics, University of Beira Interior through the project PEst-OE-MAT-UI0212-2011. This work was supported by FEDER funds through COMPETE–Operational Programme Factors of Competitiveness and by Portuguese funds through the Center for Research and Development in Mathematics and Applications (University of Aveiro) and the Portuguese Foundation for Science and Technology within project PEst-C-MAT-UI4106-2011 with COMPETE number FCOMP-01-0124-FEDER-022690.

## References

- [1]P. Bergau and D. Garbe (1989). Non-orientable and orientable regular maps. *Proceedings of Groups-Korea 1988*, Lecture Notes in Math. 1398, Springer, New York, 29-42.
- [2]A. Breda d'Azevedo and R. Nedela (2003). Chiral hypermaps of small genus. *Beitr. Algebra Geom.*, 44, 127-143.
- [3]A. Breda, A. Breda d'Azevedo and R. Nedela (2006). Chirality group and chirality index of Coxeter chiral maps. *Ars Combin.*, 81, 147-160.
- [4]A. Breda d'Azevedo, G. Jones, R. Nedela, and M. Skoviera (2009). Chirality groups of maps and hypermaps. *J. Algebraic Combin.*, 29, 337-355.
- [5]A. Breda d'Azevedo and R. Nedela (2003). Chiral hypermaps with few hyperfaces. *Math. Slovaca*, 53, 107-128.
- [6]P. R. Bryant and D. Singerman (1985). Foundations of the theory of maps on surfaces with boundary. *Quart. J. Math. Oxford, Series 2*, 36, 17-41.
- [7]M. Conder. Lists of regular maps and hypermaps up to genus 101. [Http://www.math.auckland.ac.nz/](http://www.math.auckland.ac.nz/)
- [8]D. Corn and D. Singerman (1988). Regular hypermaps. *European J. Combin.*, 9, 337-351.
- [9]D. Garbe (1969). Über die regulären Zerlegungen orientierbarer Flächen. *J. Reine Angew. Math.*, 237, 39-55.
- [10]M. Izquierdo and D. Singerman (1994). Hypermaps on surfaces with boundary. *European J. Combin.*, 15, 159-172.
- [11]D. L. Johnson (1997). *Presentations of Groups* 2nd ed.. London Math. Soc., Stud. Text 15, Cambridge University Press, Cambridge, UK.
- [12]L. D. James (1988). Operations on hypermaps and outer automorphism. *European J. Combin.*, 9, 551-560.
- [13]G. A. Jones and D. Singerman (1978). Theory of maps on orientable surfaces. *Proc. London Math.Soc.*, 37, 273-307.
- [14]G. A. Jones and D. Singerman (1994). Maps, hypermaps and triangle groups. In *The Grothendieck Theory of Dessins d'Enfants*, London Math. Soc., Lecture Notes Ser. 200, Cambridge University Press, Cambridge, UK, 115-145.
- [15]T. R. S. Walsh (1975). Hypermaps versus bipartite maps. *J. Combin. Theory, Series B*, 18, 155-163.
- [16]S. E. Wilson (1985). Bicontactual regular maps. *Pacific J. Math.*, 120, 437-451.
- [17]S. E. Wilson. Census of rotary maps. [Http://www.ijp.si/RegularMaps/](http://www.ijp.si/RegularMaps/).
- [18]S. Wilson and A. Breda d'Azevedo (2003). Non-orientable maps and hypermaps with few faces. *J. Geom. Graph.*, 7, 173-190.

## Trends and seasonality of the road accidents in Angola from 2002 to 2015

Manuel Alberto<sup>1,3</sup>, Dulce Gomes<sup>1,2</sup> and Patrícia A. Filipe<sup>1,2</sup>

<sup>1</sup>Centro de Investigação em Matemática e Aplicações, Instituto de Investigação e Formação Avançada, Universidade de Évora, Portugal

<sup>2</sup>Departamento de Matemática, Escola de Ciências e Tecnologia, Universidade de Évora, Portugal

<sup>3</sup>Instituto Superior Politécnico Internacional de Angola, Angola

**Corresponding Author:** *ad11837@alunos.uevora.pt*

### Abstract

Road accidents are one of the major causes of mortality in Angola, and it is considered the second major cause of mortality after malaria and simultaneously an epidemic [4]. In this work, we will present a characterization of road accidents in Angola from 2002 to 2015. The study is based on the databases of the Direcção Nacional de Viação e Trânsito da Polícia Nacional (DNVT/PN) and the Gabinete de Estudos, Informação e Análise do Comando Geral da Polícia Nacional (GEIA/CGPN). The variables involved in the analysis are the number of accidents, the nature of the accident, the number of injuries/deaths, the Province, the year/month, the climate among other factors. A Seasonal-Trend Loess (STL) decomposition was employed [1, 2] to decomposes a time series into seasonal, trend and irregular components using loess. Seasonal Autoregressive Integrated Moving Average (SARIMA) models, based on the Box-Jenkins principles [3], were fit in order to characterize the series behavior of the monthly rates of road accidents. We intend to make a study with the aim of obtaining an adequate statistical model and contribute to the understanding of the main determinants of road accidents and to create a document that serves as an instrument to make forecasts in the medium and long term.

**Keywords:** Road accidents, Angola, Time Series, STL, SARIMA.

### Acknowledgements

The researchers belongs to the Centro de Investigação em Matemática e Aplicações, Universidade de Évora, Project UID-MAT-04674-2013, a research centre supported by FCT (Fundação para a Ciência e a Tecnologia, Portugal).

## References

- [1]Cleveland R, ClevelandW, Mcrae J, Terpenning I. (1990) STL: A Seasonal-Trend Decomposition Procedure Based on Loess. *Journal of Official Statistics*, 6(1), pp.3–73.
- [2]Hyndman RJ, Athanasopoulos G. (2012). *Forecasting: principles and practice* Internet. Available from: <http://otexts.com/fpp/>
- [3]Shumway DRH, Stoffer PDS. (2011). *Time Series Regression and Exploratory Data Analysis. Time Series Analysis and Its Applications* (Internet). Springer New York, pp. 47–82. Available from: <http://link.springer.com/chapter/10.1007/978-1-4419-7865-3-2>
- [4]Sinistralidade Rodoviária em Angola (2014). Direcção Nacional de Viação e Trânsito da Polícia Nacional de Angola (Documento Policopiado).





## Index of authors

- A.M., Simões, *30*  
Abranches, Margarida, *50*  
Aldwihe, Ramez, *59*  
Almeida, Nicolau, *36*  
Andrade, Leonardo, *58*
- Bandeira, Luís, *5*  
Boutoulout, Ali, *25*  
Branco, João A., *6*  
Branco, Manuel B., *2*  
Braumann, Carlos A., *16*  
Brites, Nuno M., *16, 60*
- Capistrano, Gilberto, *4*  
Carapau, Fernando, *6, 47*  
Carloto, António, *62*  
Caxala, Zeferino, *49*  
Colaço, Isabel, *9*  
Conceição, Ana C., *39*  
Conceição, Ricardo, *6*  
Correia, Paulo, *6, 47*  
Costa, Sérgio Cavaleiro, *28*  
Cravo, Pedro M., *15*
- D'Avezedo, António Breda, *64*  
Dadang, Amir Hamzah., *20*  
Deuchande, Sofia, *50*  
Dias, Cristina, *1, 7, 21, 36, 38, 40, 43*  
Dias, José G., *13*  
Domingues, Luís Filipe, *13*
- El Otmani, Rabie, *14, 18*
- Faria, S., *55*  
Faria, Susana, *56*  
Ferreira, Dário, *4*  
Ferreira, Lúcia, *60*  
Ferreira, Sandra S., *4*  
Figueira, Victor, *35*  
Filipe, Patrícia A., *66*  
Fortes, Paulo, *52*
- Godinho, M. Teresa, *35*  
Gomes, A.R., *55*  
Gomes, Dora Prata, *11*  
Gomes, Dulce, *66*  
Gonçalves, A.M., *55*  
Gonçalves, M. Manuela, *57*  
Gonçalves, Pedro, *58*
- Grácio, Maria Clara, *23*  
Grilo, Helena L., *8*  
Grilo, Luís M., *6, 8, 47*
- Hajjaji, Abdeloawahed, *14, 18*  
Hayat, Zouiten, *25*
- Janeiro, Fernando, *28, 58*
- Kandoussi, Khalid, *14, 18*
- L.P., Castro, *30*
- Machado, Manuel, *66*  
Malico, Isabel, *28*  
Mateus, Dina, *45*  
Melgueira, Pedro, *60*  
Mendes, L., *26*  
Mesbahi, Oumaima, *14, 18*  
Mexia, João Tiago, *1, 4, 38, 40, 43*  
Miranda, João, *1, 21*
- Neves, Manuela, *11*  
Novais, Luísa, *56*  
Nunes, Célia, *4, 40, 43*
- Oliveira e Silva, Pedro, *52*  
Oliveira, Amílcar, *10*  
Oliveira, T.A., *26*  
Oliveira, Teresa, *10*
- Pereira, J.L., *26*  
Pinto, Henrique, *45*  
Pires, Ana M., *6*  
Portela, M. M., *3*  
Portugal, João, *52*  
Pulido-Calvo, I., *3*
- Ramôa, Sofia, *52*  
Ramos, Carlos C., *1, 5*  
Ribeiro, Andreia, *57*  
Rocha, Eugénio, *20*  
Rodrigues, Ilda Inácio, *49, 64*  
Rodrigues, Irene, *23, 60*  
Rodrigues, J., *55*  
Romacho, João, *7, 36, 38*
- Saias, José, *59*  
Sancho, Luís, *35*

Santos, Carla, *1, 36, 38, 40, 43*

Santos, J. F., *3*

Sellami, Assia, *14, 18*

Serôdio, Rogério, *49*

Simões, C., *55*

Simão, Carla, *50*

Teixeira, Ana, *50*

Teodoro, M. Filomena, *11, 50*

Tlemçani, Mouhaydine, *14, 18, 58*

Varadinov, Maria José, *7, 21, 36, 38*

Vasconcelos, Teresa, *52*

Zapata, Juan Luis García, *23*



## Local Sponsors:

